PRINCIPLES OF MECHANICS

Mehreen Nagi

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➢1: Basics of physics

Mechanics, which is the study of motion and its causes, involves measuring distances, recording time, weighing objects, and performing complicated calculations. There would be so much confusion if everyone performed these calculations differently. Assume that a group of ten students is asked to estimate the speed of a football kicked across a field. Without a standardized measurement, every student would come up with a different value.

This brings us to the importance of standardized units of measurement, which ensure that quantities are measured the same way. We have seven common units known as the SI units (international system of units). They include:

Measurement	**SI unit**
Length	Metre (m)
Mass	Kilogram (kg)
Time	Second (s)
Temperature	Kelvin (K)
Amount of substance	Mole (mol)
Luminous intensity	Candelabra
Electric current	Ampere (A)

It should be noted that all other units are made by adding either a multiple or a submultiple in front of the SI unit. For example, a millimeter is

made up of the submultiple (mili =10^{-3}) and the SI unit meter. Here's a list of multiples and submultiples that can be used to form larger/ smaller units:

NAME	SUBMULTIPLES	MULTIPLES
DECI	10^{-1}	
CENTI	10^{-2}	
MILI	10^{-3}	
MICRO	10^{-6}	
PICO	10^{-12}	
NANO	10^{-9}	
KILO		10^{3}
MEGA		10^{6}
GIGA		10^{9}
TETRA		10^{12}

i. Base units, derived units, and homogenous equations

All SI units are known as base units. They strictly comprise the seven units listed in the table above. On the other hand, derived units are made up of the seven SI units. Derived units may be expressed as a combination of SI units (kg/m^3) or as a different unit (J, N, Pa). Every single derived unit can be expressed in terms of its base units. It's important to understand how to do so:

Example 1

Force = N

= ma

= kg. ms^{-2}(Expressed in terms of base units)

Here's a harder example:

Voltage = V

= work done / charge

=J/C

= Nm/ C

= kg$m^2 s^{-2}$. $A^{-1} s^{-1}$

ii. Errors and uncertainty

In physics, we often hear the words error and uncertainty used interchangeably, but they have really precise meanings. When we say there is an "error" in our reading, we mean that our recorded/measured value is different from our true value. The amount by which our reading differs and how it differs depends on the type of error involved. Errors are classified into two main types:

1- Random error

2- Systematic error

Error: Difference between the true and measured value.

Random errors: When reading a burette in the chemistry laboratory, you align your eyesight horizontally to the bottom or upper meniscus. Imagine that the correct level of liquid in the burette is 25.0 cm^3, but you read 25.3. Next time you mistakenly read lower than where the meniscus is and record 24.8cm^3.

The readings differ from the actual value because there is a specific type of error involved: parallax error, a type of random error. It should be noted that your readings fluctuate both above and below the actual value and they vary by a different amount each time. Random errors can therefore be described as:

- Errors that cause readings to fluctuate both above and below the actual value
- Differ by a different amount each time
- Affect the precision of the reading

Systematic errors

Now, you are trying to record the diameter of a thin piece of copper wire using a micrometer screw gauge. The copper wire has a diameter of 0.08 mm but you keep on getting a reading of 0.05. The recorded reading differs by the same

amount every time you take a measurement. This is because of a zero error in the micrometer. Zero errors are a type of systematic error. It should be noted that a systematic error:

- Causes the reading to differ by the same amount
- Makes the reading fluctuate on the same side of the original reading (either above or below it)
- Affects the accuracy of the instrument.

Precision vs accuracy

Precision refers to the smallest reading that can be measured by an instrument. For example, a meter rule measures correctly to 0.5 cm, and a stopwatch has a precision of 0.01s. Similarly, a micrometer measures correctly to 0.01 mm and a Vernier caliper to the nearest 0.1 mm. The precision of a set of readings refers to the spread of those readings. It indicates the likeliness of obtaining a similar reading if the experiment is repeated. If there is a set of readings clustered together, then the readings are said to be precise. If they aren't widely spread you are likely to obtain a similar reading if the experiment is repeated.

On the other hand, if there is another set of readings where the values are far apart, then the readings are said to be imprecise.

Accuracy

Accuracy is a term used to describe the closeness of a measured value to the true reading. Readings very close to the true reading are highly accurate, and those far away have low accuracy. In practice, readings can be a combination of the two. For example, a reading can be both accurate and precise, neither accurate nor precise, precise but not accurate.

Uncertainty

We've distinguished between accuracy and precision. Accuracy is just a measure of the closeness to the true value and precision is the spread of a set of values. Then what is uncertainty?

In simple words, uncertainty is the range in which an inaccurate value is most likely to lie. This is often called the absolute uncertainty and is written with a ± sign, indicating that the true value lies within a certain amount (as indicated by the absolute uncertainty) either above or below the measured value.

Fractional uncertainty is simply the absolute uncertainty/ measured value. If the absolute uncertainty is 'x' and our reading is 'y', then the fractional uncertainty is given by:

$\frac{x}{y}$

Multiplying the fractional uncertainty by 100 gives us the percentage uncertainty.

Combining uncertainties

When we are dealing with calculations involving two or more measurements, we combine the uncertainties to find the uncertainty in the final value. The following two rules are used:

- If two quantities are added or subtracted, the uncertainties are added. This can be expressed as:

Quantity X with its absolute uncertainty = (X ± x)

Quantity Y with its absolute uncertainty = (Y ± y)

Z= X * Y

Then the uncertainty in Z is given by = x + y

2. If two quantities are being multiplied or divided, then the percentage uncertainties are found first, then they are added and the absolute uncertainty is found.

$\frac{X}{Y}$

Total uncertainty = $\frac{x}{X} * 100 + \frac{y}{Y} * 100$

3. The power a quantity is raised to is always multiplied by the uncertainty in that quantity. For example

$volume = \pi r^2$

$r = (r \pm \% r)$

Uncertainty in volume is given by= 2r%

Example:

A cuboid with a square base has a base length of (15.0 ± 0.5) cm and a height of (25.0 ± 0.1) cm. Calculate the volume of the cuboid with the absolute uncertainty in it.

Volume = base area * height

= 15 * 15 * 25 = 5625

Uncertainty in base length = $\frac{0.5}{15} * 100 = 3.3\%$ **= 2 * 3.3 = 6.6 %**

Uncertainty in height = $\frac{0.1}{25} * 100 = 0.4\%$

Total uncertainty = 5 %

Absolute uncertainty in volume =

$\frac{5}{100} * 5625 = 281.25$

Volume = (5625.00 ± 281.25)

➢ Chapter 2: kinematics

If an object changes position relative to a reference point, the object is in motion. Kinematics is the study of a body’s motion. An important thing to note is that all the aspects of

motion are relative to time. For example, the change in displacement relative to time is a body’s velocity. Similarly, the change in velocity with respect to time is the acceleration. The change in acceleration with respect to time can be described as the second derivative of acceleration ($\frac{d^2 v}{d x^2}$¿. This tells us the rate of change of acceleration.

I. Distinguishing between distance vs displacement and speed vs time.

Imagine two people standing at opposite ends of a field. If one of them has to move towards the other, he or she can either walk in a straight line or a curved path, changing directions multiple times before reaching the other end. The shortest possible distance the person would have covered in a straight line is known as **displacement**. The total length covered regardless of the direction taken is the **distance.**

Displacement takes into account the direction of a moving body whereas distance does not. Likewise, speed is the rate of change of distance and velocity is the rate of change of displacement.

Note: Velocity and displacement have signs. Any direction (usually the right is assigned a positive sign and left a negative sign). If a body changes direction, as in the case of a body moving towards its starting point, the velocity and displacement both change direction.

ii. Uniform, instantaneous, and average velocity

Earlier, we defined velocity as the rate of change of displacement. This rate of change can either be steady or variable. When a body is covering equal distances/ displacements in equal intervals of time, it is traveling with uniform velocity. If the rate differs, which is usually the case, the velocity is variable. In such a case, it would be more useful to find the average velocity. The average velocity is simply the total displacement divided by the total time taken. This is given by:

$$\frac{total\,displacement}{total\,time\,taken}$$

Moreover, if we are interested in finding the velocity at any given instant, we can calculate the instantaneous velocity by drawing a tangent on that particular point on the graph.

iii. Acceleration, deceleration, and equations of motion

Acceleration is a general term used to describe a change in a body's velocity. Since acceleration refers to an increase as well as a decrease in velocity, it is important to use a negative sign when talking about a decrease.

Deceleration, on the other hand, specifically talks about a decrease in acceleration. Therefore, it is not necessary to put a negative sign with it.

$$a=\frac{v-u}{\Delta t}$$

Equations of motion

There are five equations of motion derived from the basic formulas of acceleration and average speed. These equations can only be used if the acceleration is constant and the body is traveling in a straight line.

1st equation of motion

$v = u + at$

2nd equation of motion

$s = \frac{1}{2}(u + v)t$

3rd equation of motion

$s = vt - \frac{1}{2}at^2$ (substituting u from 1 equation into equation 2)

4th equation of motion

$s = ut + \frac{1}{2}at^2$ (substituting v from equation 1 into 2)

5th equation:

$2as = v^2 - u^2$

iv. Free Fall vs Air resistance

Everything near the Earth’s surface falls at a constant acceleration of 9.8ms^{-2} when air resistance is negligible. This value is the same for all bodies, regardless of the mass, shape, size, or height from which the body is thrown to the surface. The rationale behind the free-fall concept is the fact that a uniform force of 9.8 N acts on every kilogram, causing a body’s velocity to increase by 9. 8ms^{-2}. However, the situation changes under air resistance. The conditions on Earth are not ideal; therefore, it is impossible to ignore the effects of air resistance

How do different bodies fall under air resistance?

If you drop two objects of different masses a pencil and a heavy book from a considerable height above the ground, you’ll notice that the

heavier object lands first. This happens because of air resistance, which opposes the motion of moving bodies. When air resistance acts on a body, it balances out the downward pull of gravity, reducing the resultant force in the downward direction. When the resultant force decreases, so does the acceleration.

The book has more weight than the pencil. While air resistance acts on both objects, a relatively large amount of air resistance is required to fully balance the book's weight as compared to that of the pencil. This means that the pencil reaches terminal velocity (discussed later) within the first few seconds and falls with constant speed, whereas the book continues to accelerate. Since the book's speed is increasing, it covers the same height in lesser time. Acceleration decreases the total travel time. It is important to note that all other factors have been held constant when evaluating the relationship between mass and travel time in the presence of air resistance. If the two objects were dropped from different heights, the results would have been different.

Surface area and air resistance

Air resistance increases with surface area. It makes sense that the more a body is exposed to air, the greater the resistive force. This means

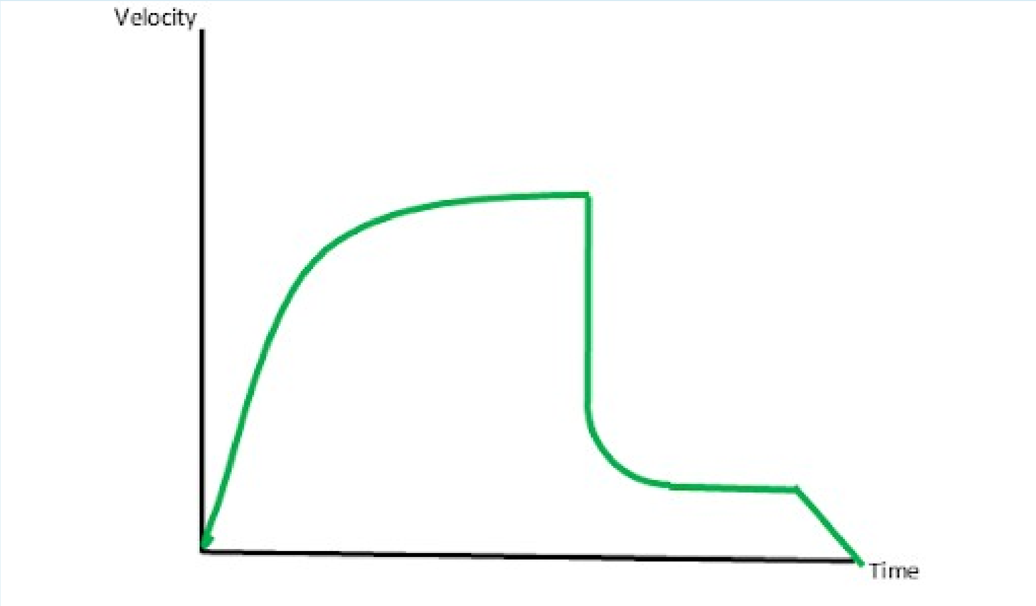
that objects with a larger surface area will reach terminal velocity earlier. However, objects with more surface area have large masses, which means that they would continue to accelerate. Usually, the effect of mass outweighs that of surface area, unless the surface areas are different and mass does not differ significantly.

Terminal velocity

The maximum speed reached by a body falling under air resistance is known as terminal velocity. When a body reaches terminal velocity, the resultant force becomes zero and it stops accelerating. The speed it attains remains constant for the rest of its fall.

The best example is of a parachutist. In the first few seconds of his or her fall, the parachutist accelerates at 9.8ms^{-2} (fall of free value). But as the speed increases, the air resistance also increases. The air resistance, which is acting in the direction opposite to the parachutist's motion, partially balances the downward force of gravity. This decreases the resultant acting in the downward direction. As the resultant force decreases, the acceleration also decreases. Now, the parachutist falls with decreasing acceleration; in other words, the speed is increasing but at a decreasing rate. Eventually, the air resistance becomes large enough to

completely balance the parachutist’s weight. He or she has finally reached terminal velocity



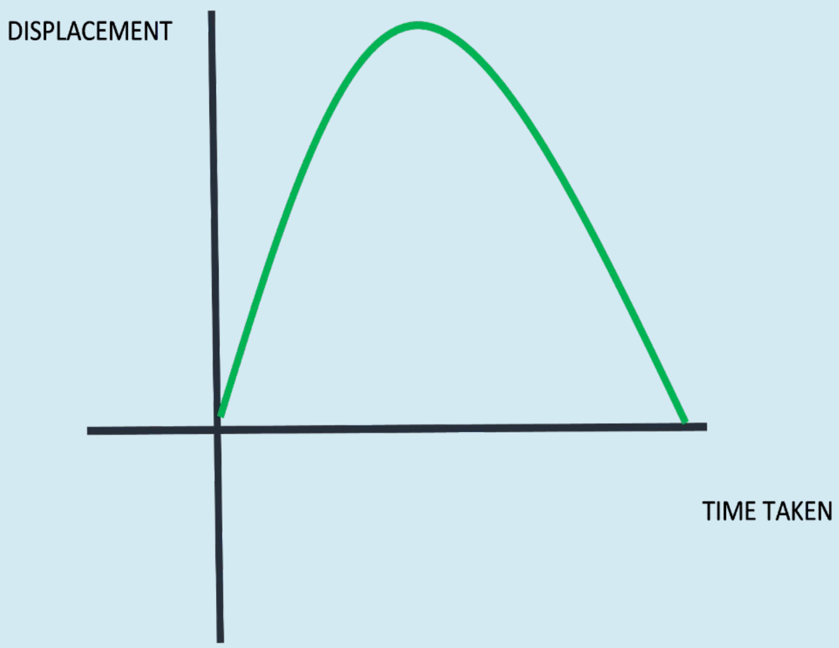
The graph shows how the speed of a parachutist varies with time. The acceleration is greatest at the beginning of the fall.

However, if the skydiver decides to open his or her parachute, rapid deceleration takes place. This happens because there is a rapid spike in air resistance due to increased surface area.

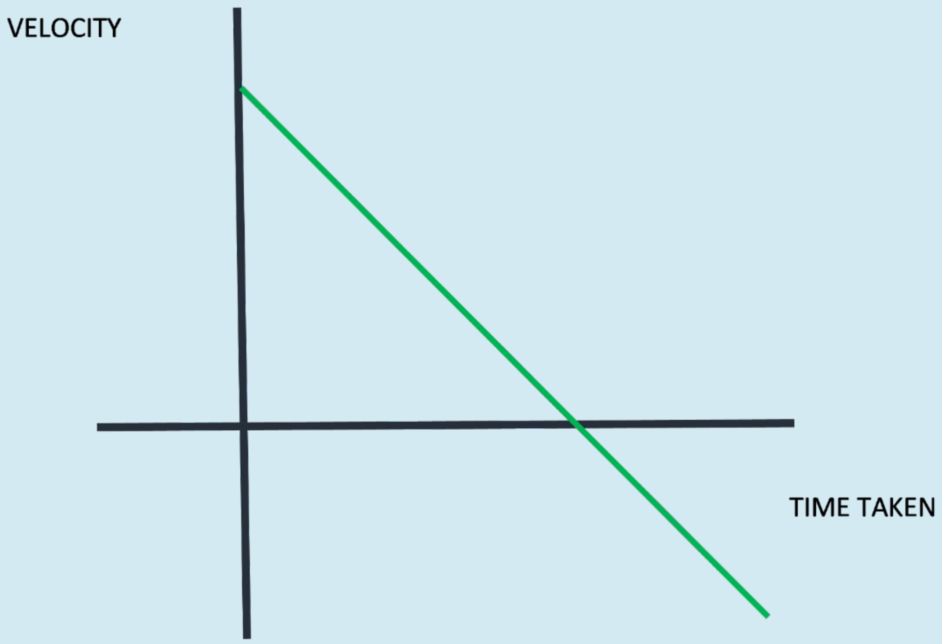
Vi. Up and down motion.

As discussed earlier, both velocity and displacement have directions. If we throw a ball vertically upwards, the velocity decreases. This is due to the downward pull of gravity. The velocity keeps on decreasing until the ball

reaches its maximum position, where the velocity is zero. It then starts falling in the opposite direction. Since the ball is moving in the same direction as the pull of gravity, its velocity increases. But remember that the direction has changed and the sign of velocity must change too. So we take velocity as negative in the opposite direction. But the value of g remains constant at – 9.8 throughout. So when the ball is moving upwards, the value of g is negative but velocity is positive. Both have opposite signs, which makes sense because velocity is decreasing. Then when the ball moves downwards, g and v have the same sign, meaning that velocity is increasing.

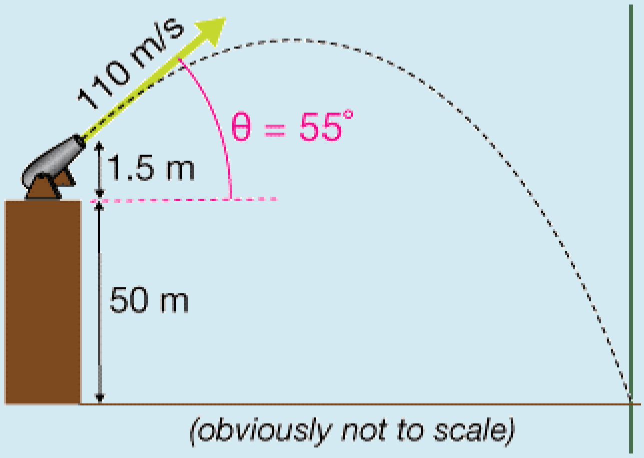


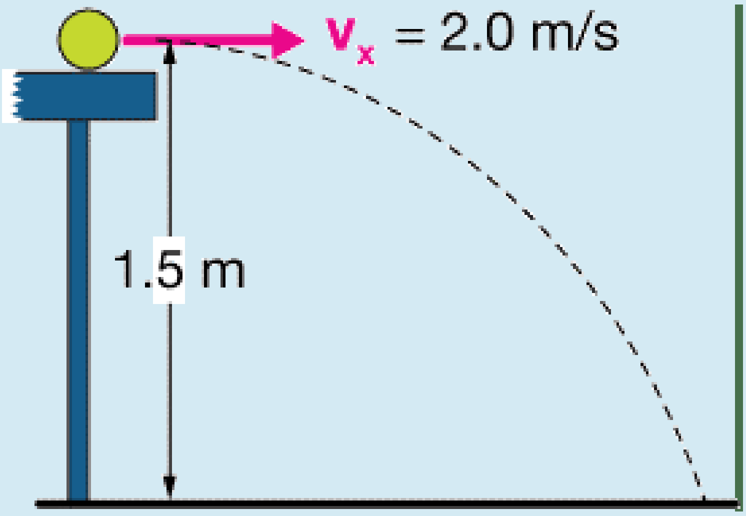
Since the ball moves in the opposite direction, the displacement starts decreasing after reaching its maximum value. The overall displacement is zero.



The graph shows that velocity decreases when the ball is thrown upwards. At its maximum height, the velocity is zero, which is shown by the point where the graph crosses the x-axis. Then when the velocity gets negative, the ball is now moving in the opposite direction. The value of g is negative, which shows that the ball is accelerating in the opposite direction

VII. Projectile motion





Projectile motion occurs when a body travels in two directions simultaneously. There are two possible directions of motion: horizontal and vertical. Most objects moving around us strictly follow one of the two directions: it is either horizontal or vertical. For example, a train has horizontal motion only. A stone dropped from the top of a terrace has vertical velocity only. But it is possible for a body to have two-dimensional motion.

Let's take the example of a ball thrown at a certain angle to the horizontal. This ball travels

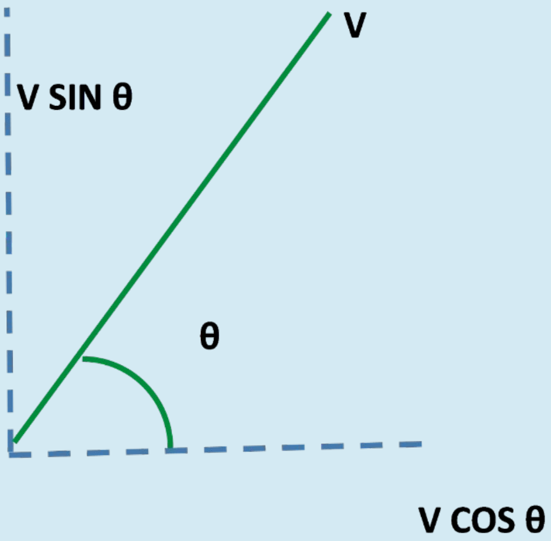
horizontally and vertically at the same time. Assuming there is no air resistance, the ball has a constant horizontal velocity and decreasing vertical velocity. As the ball moves further up, the value of free fall (which is taken as – 9.8 ms^{-2}¿ causes the velocity to decrease uniformly. However, since there is no force acting on the ball in the horizontal direction, its velocity remains constant.

The equations of motions discussed earlier can be used to calculate the distance covered in both directions. We know that when the displacement is maximum, the velocity is zero. Using this information, we can find an equation for the maximum vertical displacement.

S= $\frac{1}{2}vt$ + at^2 (input v =0)

H = at^2

The horizontal displacement is simply the product of the horizontal velocity and time. If we are provided with an angle in the question, then the horizontal velocity is U cos θ



Moreover, we can also find an expression for the time taken to reach the maximum height:

$$V = u + at$$

Input v= 0

The horizontal component of velocity = U sin θ

$$a = -g$$

Our final expression for time becomes,

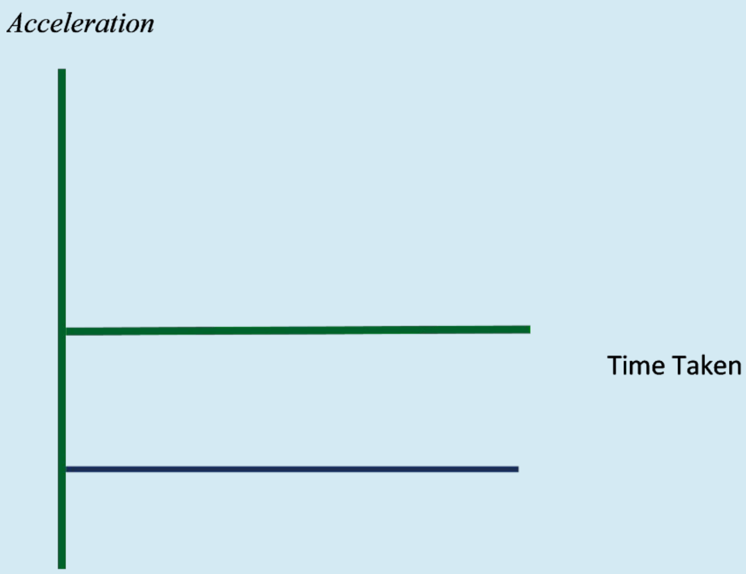
$$T = U \sin \theta / g$$

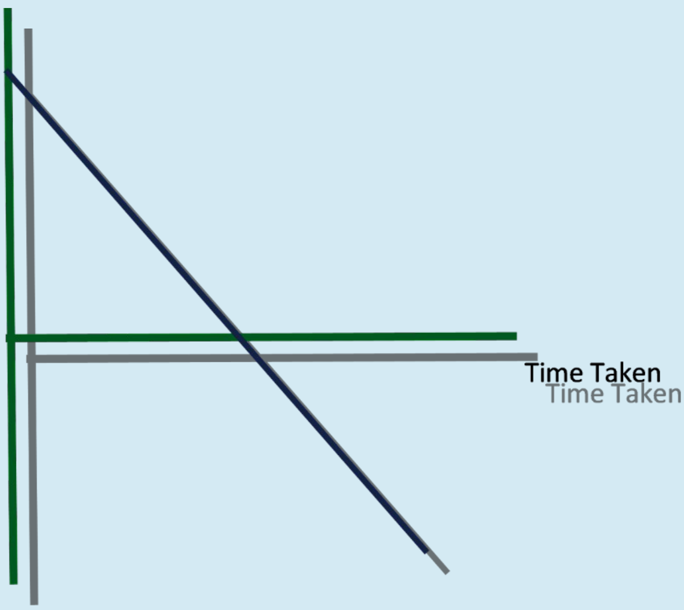
For the total time taken, we multiply the expression by 2.

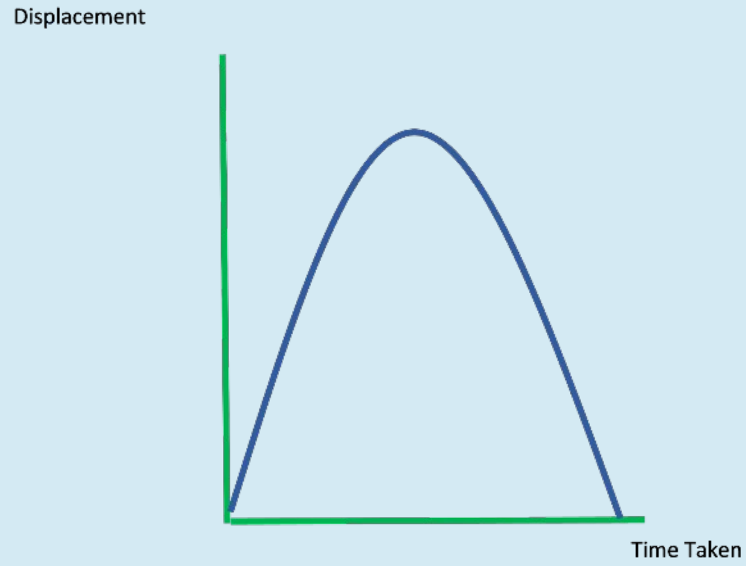
Graphs of projectile motion

The following graphs show how the acceleration, displacement, and velocity vary when

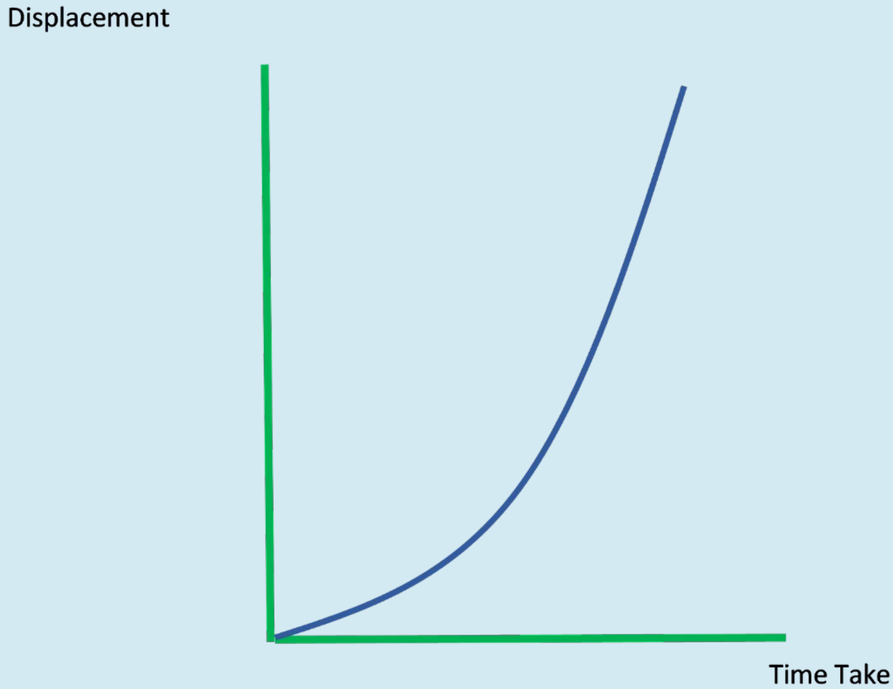
i. An object is thrown upwards

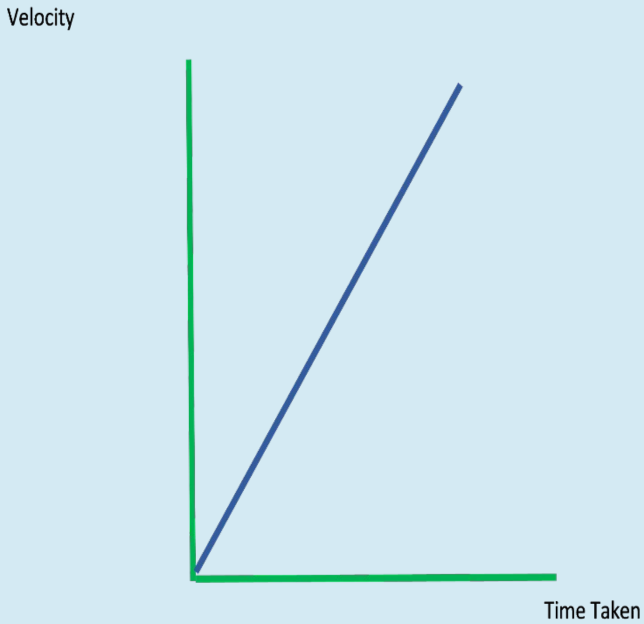


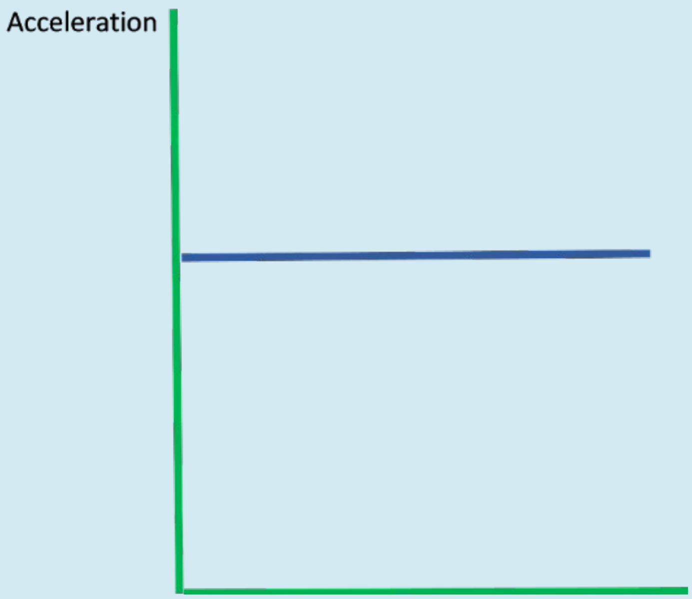




ii. An object is thrown downwards




Velocity
Time Taken

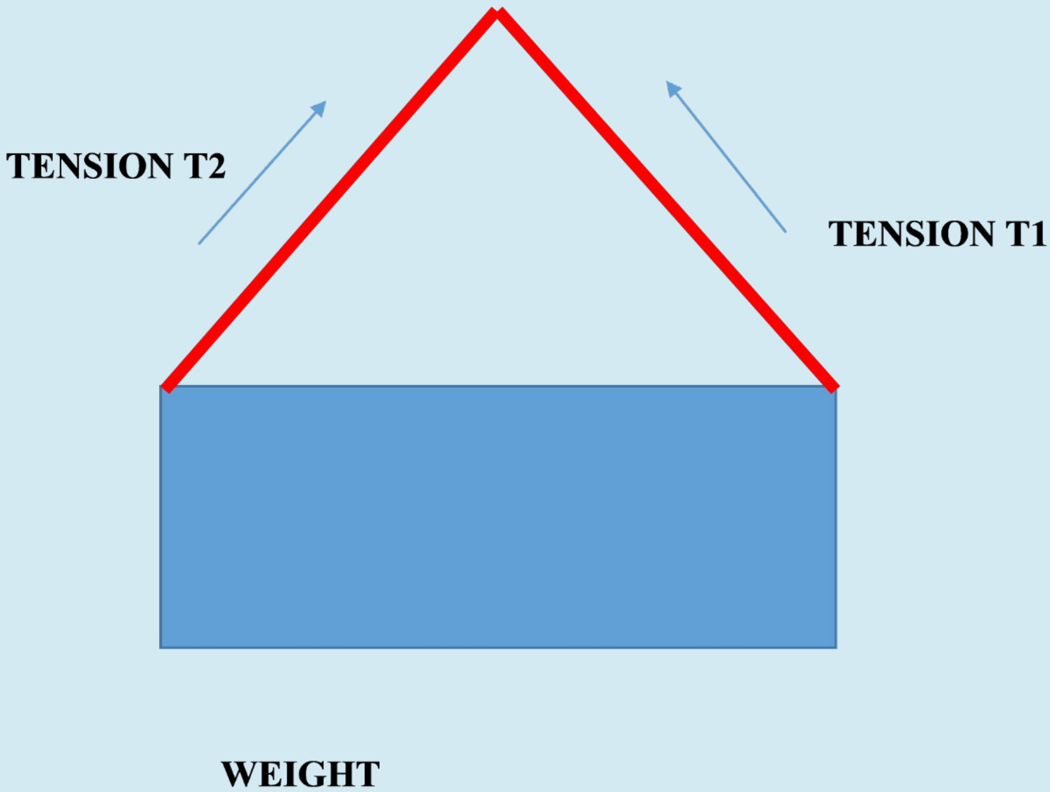

Acceleration

➢Chapter 3: Dynamics

A force is defined as a push or a pull. Forces can change the shape, size, volume, direction, velocity, and acceleration of objects but they cannot change matter and density.

The main forces we see around us include contact forces (which always act perpendicular to the surface), air resistance, upthrust (the upward force exerted by water), thrust (forward driving force on engines), and drag, which is a force that resists motion.

i. Balanced, unbalanced forces, and Newton's Laws



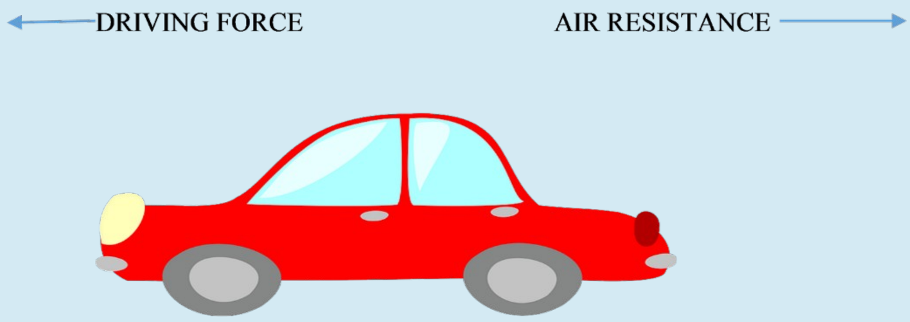
TENSION 1 AND TENSION 2 ARE EQUAL TO THE WEIGHT. HENCE, TH FRAME IS IN EQUILIBRIUM

Multiple forces can act on a body in different directions. For example, a driving force might be propelling a body forwards and air resistance might be opposing its horizontal motion. In addition, the weight would be acting downwards and the contact force exerted by the ground on the body would be pushing upwards. But since all these forces are acting on the same body, they all have a combined effect on it. This combined effect is given by the resultant force. *The resultant force is the algebraic sum of all the forces acting on a body*.

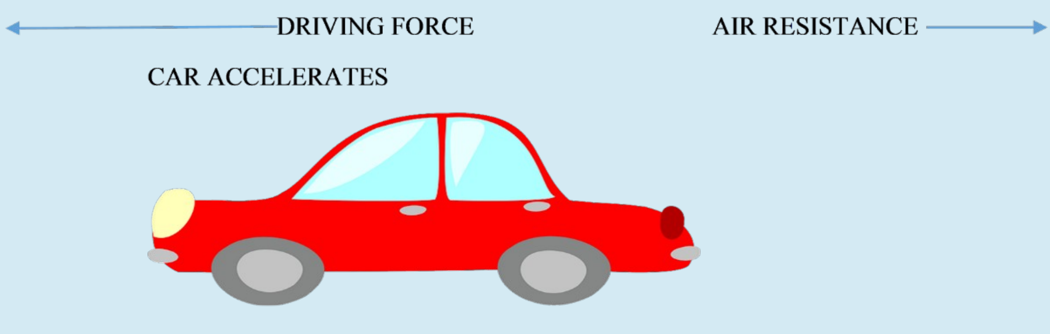
It follows that if the resultant force acting on a body is zero, then the body is balanced. A balanced state results in two possible situations:

i. The body is at rest
ii. The body is traveling at a uniform speed

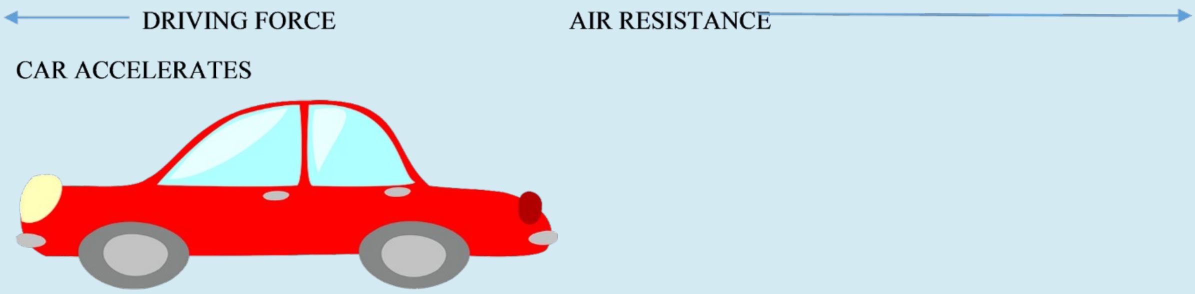
- **DRIVING FORCE = AIR RESISTANCE**
- **RESULTANT FORCE = 0**
- **EITHER UNIFORM VELOCITY OR REST**



- **DRIVING FORCE > AIR RESISTANCE**
- **RESULTANT FORCE = FORWARD DIRECTION**
- **ACCELERATES**



- **DRIVING FORCE < AIR RESISTANCE**
- **RESULTANT FORCE = BACKWARD DIRECTION**
- **DECELERATES**



This relationship between balanced forces and acceleration is summarized by newton’s first law: *A body will continue at its state of rest or uniform motion unless a resultant force acts on it.*

In other words, a body cannot deviate from its state of motion without the application of a resultant force.

While newton’s first law deals with balanced forces, the second law delves deeper into the connection between resultant force and acceleration.

The second law states that the acceleration of a body is proportional to the resultant force acting on it.

A body can only accelerate if a resultant force acts on it. This principle is the basis of Newton's second law of motion. Whether a body accelerates from rest or it undergoes an increase or decrease in its velocity, a resultant force acts on it. The relationship between force, mass, and acceleration is given by:

$$F = ma$$

The force itself is a product of the acceleration and a constant, which is the mass. The relationship can be surmised as: a force acts on a given mass, producing an acceleration. From the above equation, the following relationships can be drawn:

$$F \propto m \qquad\qquad F \propto a$$

This means that for two bodies of constant acceleration, the body with more mass requires more force. Imagine two cyclists traveling along a level road. Cyclist A has 2 people sitting at the back. Both cyclists are accelerating at $5 ms^{-2}$, but since cyclist A has thrice the mass of cyclist B, he exerts triple the amount of driving force.

On the other hand, if two cyclists have the same mass, then the one that cycles more rigorously will have greater acceleration. The acceleration increases with an increase in driving force.

Newton’s third law of motion deals with forces acting on different bodies. The law states that if body A exerts a force on body B, then body B exerts an equal and opposite force on body A. these forces are

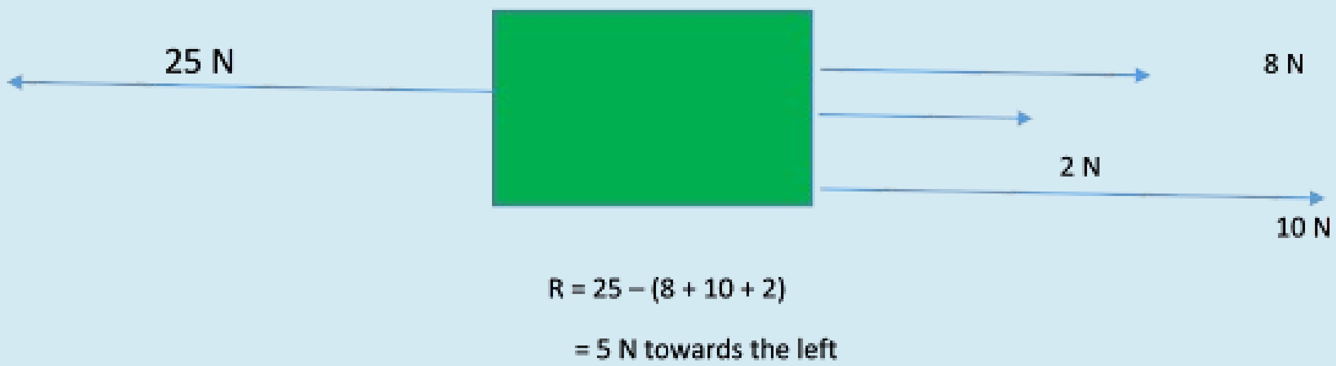
- Acting on different bodies
- Equal in magnitude
- Opposite in direction

Since these forces act on different bodies, they cannot cancel out each other. Moreover, if one of the two forces in the action-reaction pair acts on a very large mass, its effect might not be visible. This happens when we push against the surface of the earth. Our applied force is so trivial compared to the Earth’s inertia that nothing is felt.

➢Chapter 4: Forces and vectors

A vector quantity has a magnitude as well as a direction. Scalar quantities have magnitude only. The direction of a vector doesn’t have to be perfectly vertical or horizontal: it can act at varying angles to the horizontal. As in the case of multiple forces acting on a body, the resultant is the algebraic sum of the forces that are parallel. But if the forces are not parallel, then we can find the resultant by either using the parallelogram method or the head to tail

method.

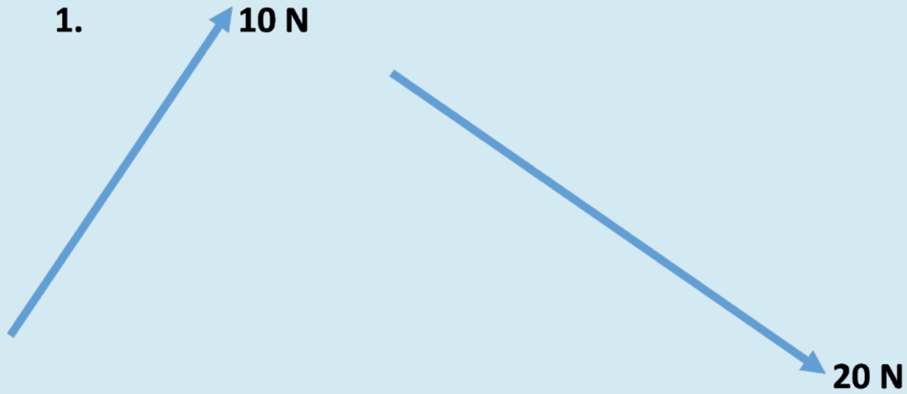


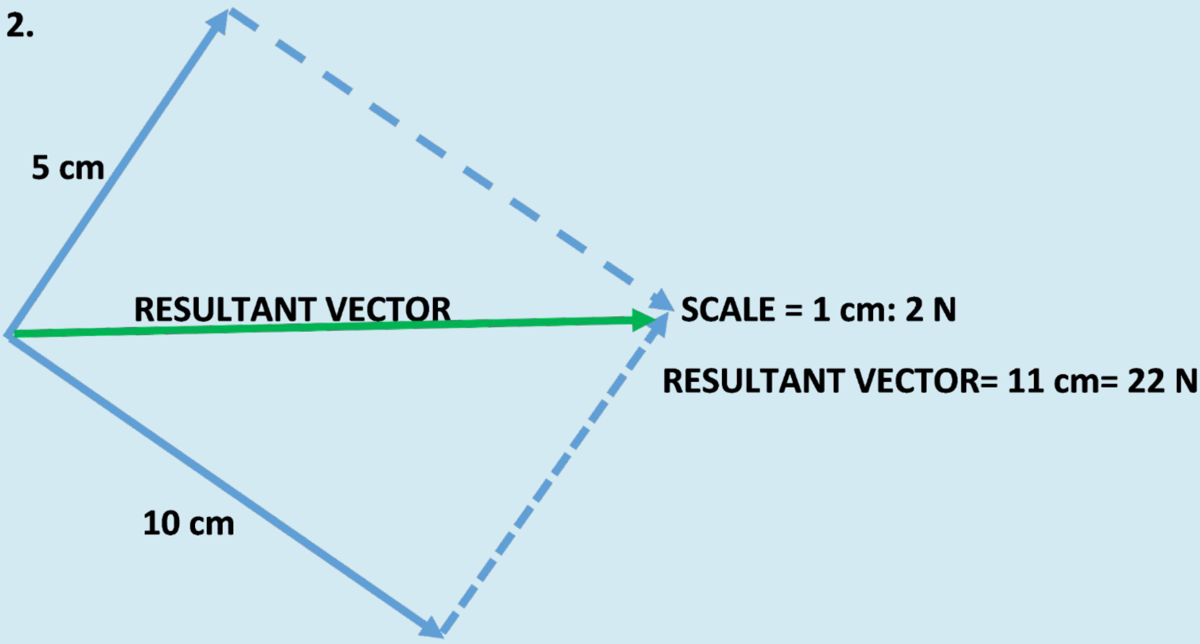
i. Composing vectors

Parallelogram method

In this method, both vectors should have the same starting point. One vector is drawn according to a suitable scale and then the second vector is drawn from the same starting point as the first one. Then a parallelogram is completed. The diagonal of the parallelogram gives us the resultant vector. To determine the magnitude of the vector, simply measure the

length of the diagonal and multiply with the chosen scale. The direction can be given by measuring the angle between the diagonal and one of the two vectors.





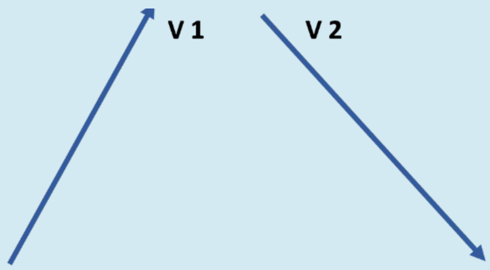
Pythagoras' theorem

Another method of finding the resultant vector is to use the Pythagoras' theorem. Note that this method only works if the two vectors are perpendicular to each other. Pythagoras'

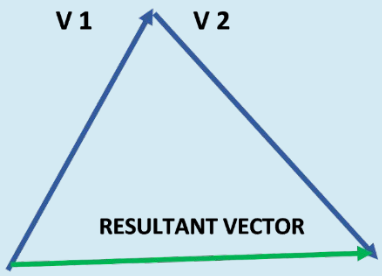
theorem is useful in situations where several forces are acting in the horizontal and vertical directions. After finding the resultant in the horizontal direction and the vertical direction, the final resultant force can be found by the Pythagoras’ theorem.

Head to tail

This method involves joining the arrow of one vector to the tail of the other. Completing the third side gives us a triangle. The magnitude and direction can then be evaluated.



2.

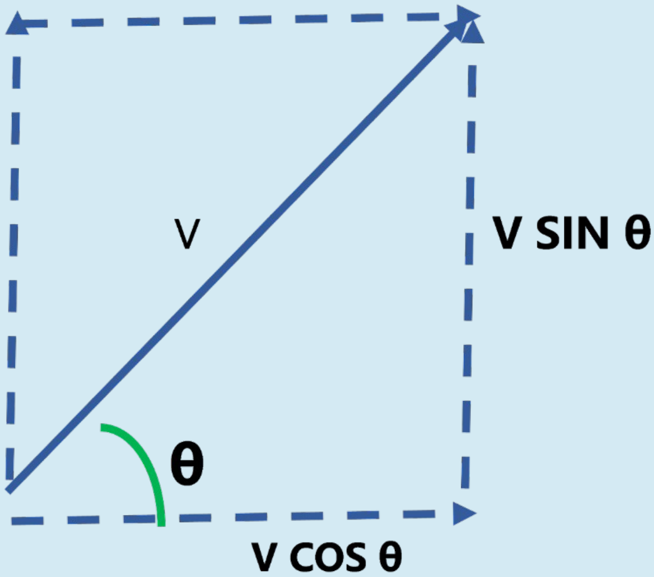


Closed triangle

A closed triangle is used to show a situation where the resultant force on an object is zero. In other words, a closed triangle shows a state of equilibrium. If each arrowhead is joined to a tail, then three forces form a closed triangle.

ii. Resolving vectors

Resolving is the exact opposite of composing. Composing a vector involved combining two or more vectors to give a resultant vector. Resolving, on the other hand, is breaking down a resultant force into its horizontal and vertical components.



The above diagram shows a right-angled triangle. Using simple trigonometry, the

horizontal component becomes F cos θ and the vertical component becomes F sin θ.

The horizontal component of force = F cos θ

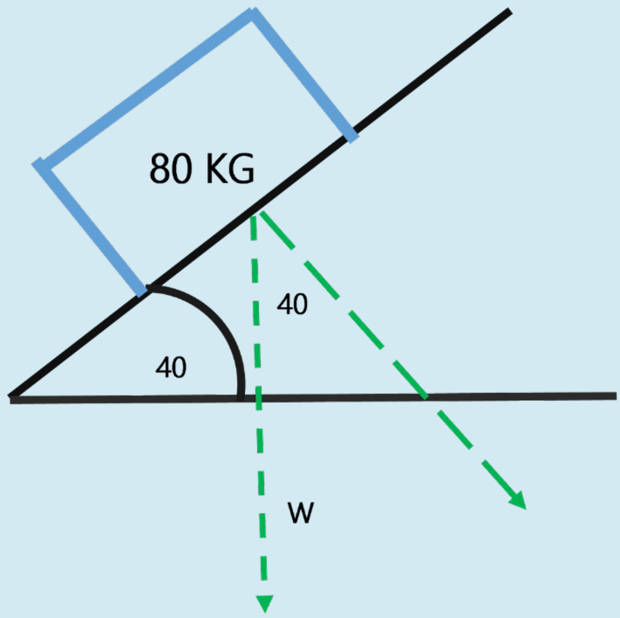
The vertical component of force = F Sin θ

Resolving the components of weight

Weight is a vector, so it can be split into two perpendicular components. Just like the above example, the component into the slope is always equal to W cos θ and the horizontal one is W sin θ.

Moving up and down slopes.

The following diagram shows that the angle the slope makes with the horizontal is equal to the angle between the weight and the vertical component of weight. The derivation is not important, but this rule should be memorized for calculations.



Moments

- The moment of a force, also known as torque, is a measure of the turning effect of a force.
- Moment is always calculated about a fixed point known as the Pivot.
- It's measured by multiplying the perpendicular distance from the line of action of the force to the pivot
- Moment = force ✖ perpendicular distance from line of action of the force
- Moment depends on:

1. Size of the force: the greater the magnitude of the force, the more the moment.
2. Perpendicular distance: the more the force is away from the pivot, the greater the

moment. This explains why it's easier to open a nut using a longer spanner.

CALCULATING MOMENTS

- If the force acts at a 90-degree angle, then we simply multiply the force and perpendicular distance

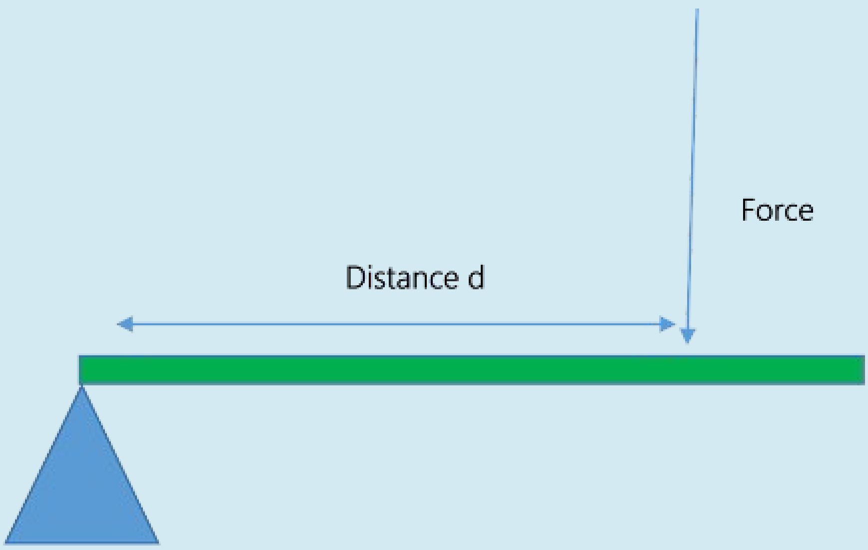


Diagram 1: The force is at a 90-degree angle, so multiply the two together

- If the force is not perpendicular, then we can either find the vertical component of the force that is perpendicular to the lever and multiply by distance 'd' (diagram 2) or we can calculate the perpendicular distance from the pivot to the force and multiply with the force (diagram 3).

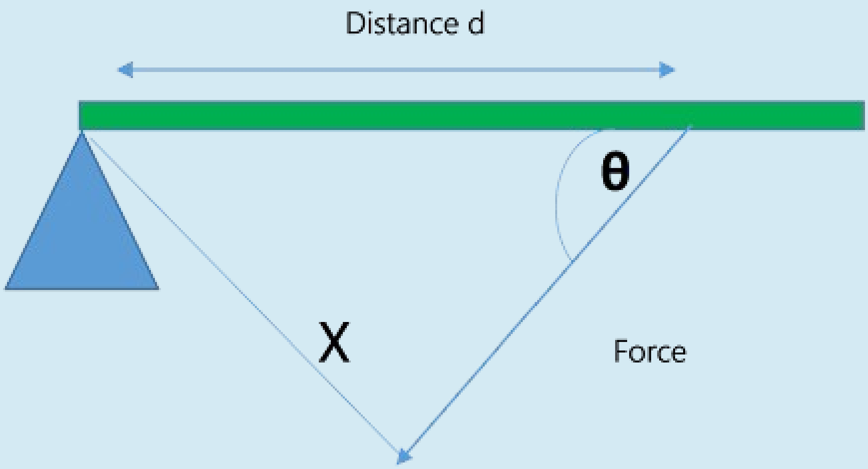


Diagram 2

- *Method 1: Calculate the perpendicular distance X ;*

$$\boldsymbol{X = d \sin\theta}$$
$$\boldsymbol{moment = F \times d \sin\theta}$$

- *Method 2: Calculate the vertical component of the force;*

$$\boldsymbol{vertical\ component = F \sin\theta}$$
$$\boldsymbol{moment = d \times F \sin\theta}$$

EQUILIBRIUM STATE

- For a body to be in equilibrium, the following conditions must be fulfilled :

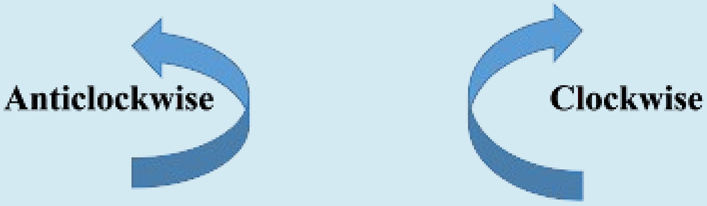
1. The resultant force must be zero
2. The resultant moment must be zero

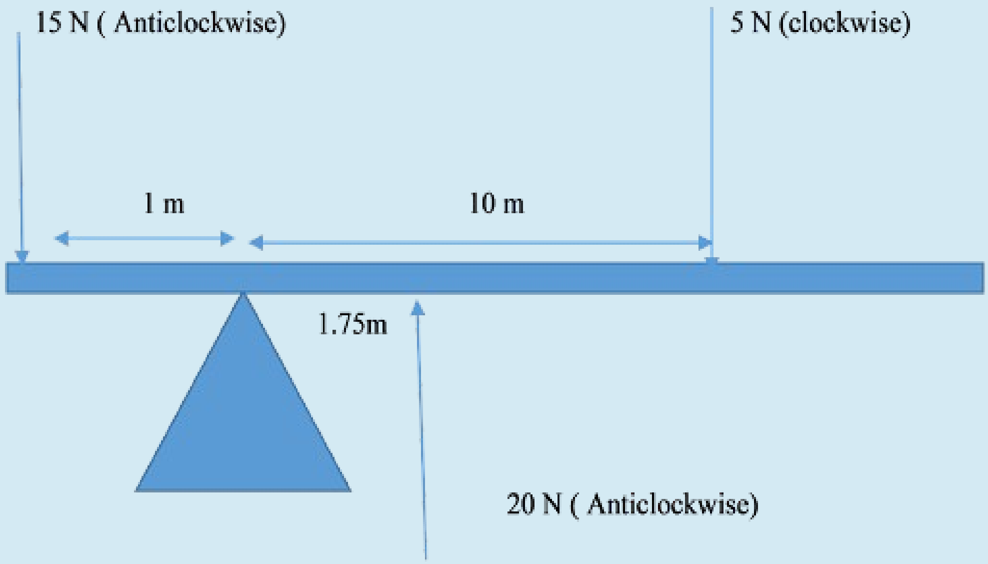
- This brings us to an essential principle about moments:

The principle of moments states that for the body to be in equilibrium, the total

clockwise moments should be equal to the total anticlockwise moments about a point.

- Note: The easiest way to determine whether a moment is clockwise or anticlockwise is to simply match its movement to that of a clock needle.





The diagram shows that ruler is in equilibrium:
Total upward force = Total downward force
20 N = 20N

Sum of anticlockwise moments = sum of clockwise moments
$(20 \times 1.75) + (15 \times 1) = 5 \times 10$
$50\ N = 50\ N$

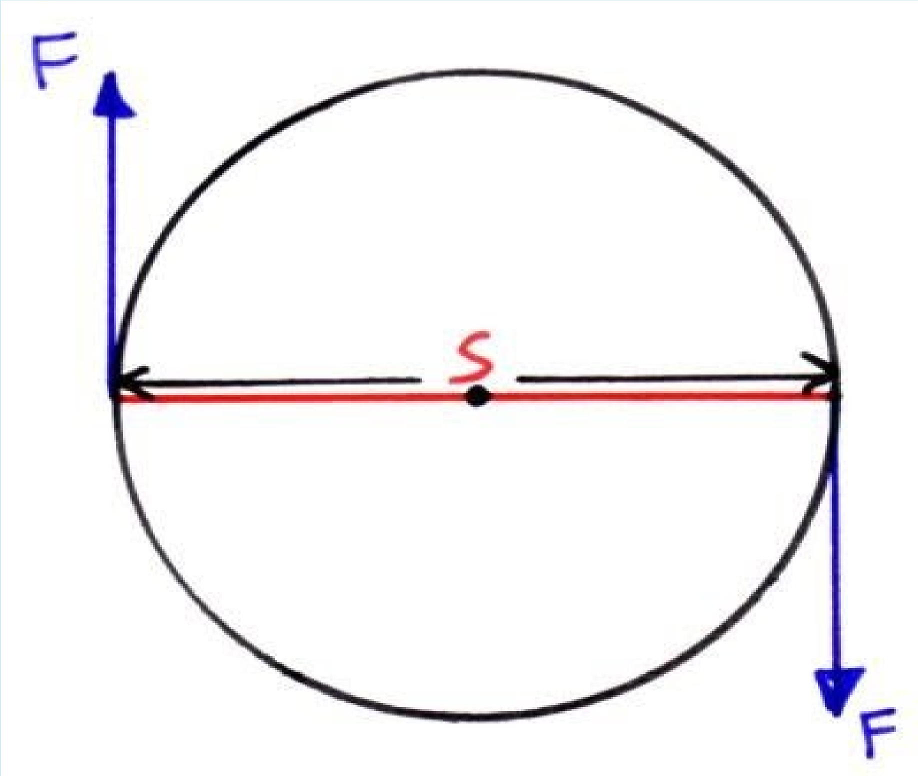
Turning effect of a couple

- Two forces that are equal in magnitude and opposite in direction, and act on the same body at different points to produce rotation are known as a couple.
- To form a couple, the forces have to be:

1. Equal in magnitude
2. parallel but moving in opposite directions
3. At a distance d from each other

- The torque of a force is calculated by multiplying one of the forces with the distance between the two

Torque of a couple = one of the two forces × perpendicular distance between them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➢ Chapter 5: Momentum and Collisions

Imagine sitting on a rollercoaster that's moving downward at a high speed. As the speed of the rollercoaster increases, the rollercoaster gains momentum.

Momentum is a term used to describe how fast a body is moving.

Linear momentum, which is a vector quantity, is the product of a body's mass and velocity.

$$P = mv$$

The unit of momentum is $kg\,ms^{-1}$. From the equation above it can be seen that momentum varies directly with mass and velocity. This means that heavier bodies have more momentum. Similarly, the higher the speed of a body, the more its momentum.

Momentum has a direction

Since momentum involves velocities, it does take into account directions. A body can either have rightward or leftward momentum. Signs are used to indicate the type of momentum. Momentum in one direction is assigned a positive sign and in the opposite direction a negative sign. When a body changes direction,

the sign of momentum also changes. For example, a ball traveling towards the right has positive momentum of 50 kg ms^{-1}. when the ball collides with a wall and bounces back, it loses rightward momentum and gains leftward momentum (in the opposite direction). The loss in momentum is equal to the gain in momentum. That is why the momentum of the system is constant.

i. Law of conservation of momentum

The momentum of bodies interacting in a closed system cannot be destroyed but can be transferred from one body to another. The idea of a closed system is important because it provides ideal conditions where the bodies are completely free of external forces. In a closed system, the bodies do not have additional acceleration due to no outside forces. When the velocities change, it always corresponds with the mass of the body (heavier mass undergoes a small change in velocity and lighter masses accelerate more). The total momentum, therefore, remains constant.

The law of conservation of momentum states that the total momentum of bodies in a closed system is constant. In other words, the net change in momentum is zero.

Let’s the example of two cars of masses M_1 and M_2. The cars have velocities v_1 and u_1. So the total momentum before the collision is:

$$= m_1 v_1 - m_2 u_1$$

Now the cars collide but continue moving in their initial directions with different velocities given by v_2 and u_2. So the total momentum after the collision is

$$= m_1 v_2 - m_2 u_2$$

Since the sum of momentum is constant, the final equation becomes:

$$m_1 v_1 - m_2 u_1 = m_1 v_2 - m_2 u_2$$

ii. Relative velocity

Relative velocity is a useful concept to describe how the velocity of a moving body changes from a specific viewpoint. Let’s imagine that a motorcyclist and truck driver are traveling in the same direction. The motorcyclist is traveling at 25 m/s while the truck driver has a speed of 15 m/s. If the truck driver is asked to describe how fast the motorcyclist is traveling, he or she would say 10 m/s, since the truck driver is covering 15 m/s (10 m less than the motorcyclist).

Now, look at the situation from the motorcyclist's perspective. Since he is moving 10m/s faster than the truck, he would say that the truck appears to move 10m/s backward. Thus, he'd say that the velocity is -10 m/s.

We can summarize the concept of relative velocity by saying that the relative speed of approach is equal to the speed of separation.

If we look at it from the truck driver's perspective, the relative velocity is positive:

$= v_2 - u_1$

$= 25 - 10 = 15$ m/s

If we look at the velocity from the motorcyclist's perspective

$= u_2 - v_1$

$= 10 - 25 = -15$ m/s

iii. Types of collisions

It is important to distinguish between elastic and inelastic collisions. In an elastic collision, the bodies move separately after colliding. In such a collision, both the kinetic energy and the total momentum is conserved. However, when bodies stick together after colliding, it is known as an inelastic collision. During inelastic collisions, bodies rub against each other, so work is done against friction. This results in the release of

thermal energy, so the total kinetic energy is not conserved. The momentum, however, is conserved. The differences in energy between the two collisions can also be described in terms of total energy.

In an elastic collision, the total energy is conserved. In an inelastic one, the energy is not conserved.

Elastic collisions and relative velocity

Consider two cars moving towards each other with velocities u_1 and v_1 respectively. Since they are traveling in opposite directions, one velocity is positive and the other is negative. The relative speed of approach is the speed measured about another body. Someone watching the two cars measures the relative speed of approach by adding the two velocities. This is because the distance between the two cars reduces as a result of each velocity. Assuming that V_1 is negative, the relative speed of approach is given by

$$= u_1 - (-v_1)$$

When the two cars collide and move off in opposite directions, the velocities reverse signs. Now, u_2 is negative. To find the sum we write the velocities the other way

$= v_2 - (- u_2)$

This is known as the **relative speed of separation**.

In an elastic collision, the relative speed of the approach is equal to the relative speed of separation.

The relative velocity of two bodies moving in the same direction is given by subtracting the two (as discussed earlier). Since two bodies moving in the same have positive signs, the same formula applies. You can simply subtract one from the other, taking one body as the reference point.

iv. Linking momentum to newton's laws

When a basketball hits the ground, it loses its momentum. The momentum lost is transferred to the ground, which in turn transfers momentum to the ball in the opposite direction. The change in momentum is known as an impulse. Impulse is given by

$$= mv - mu$$

The rate of change of momentum, or the impulse per unit time, is proportional or equal to the resultant force acting on a body. The resultant force, which is given by F= ma can be expressed in terms of momentum.

$F = ma$

$¿ m \frac{v-u}{t}$

$¿ \frac{mv-mu}{t}$

= rate of change of momentum.

Therefore, newton's second law can be written as

The resultant force is equal to or proportional to the rate of change of momentum

NEWTON'S THIRD LAW AND MOMENTUM

The third law states that if two bodies interact, the force exerted by each body on the other is equal and opposite in direction. This can be expressed as:

$$F_X = -F_y$$

We know that each force can be expressed in terms of the rate of change in momentum. This gives us:

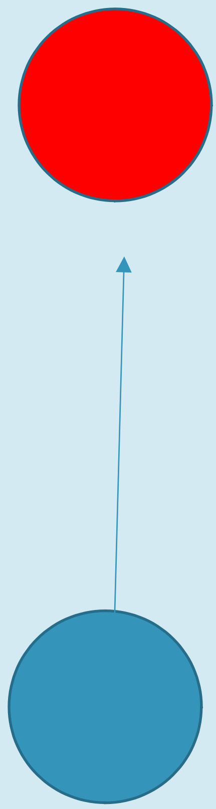
$$m_x\left(V_X - U_X\right) = -m_y\left(V_y - U_y\right)$$

From the above equation, we can deduce that the momentum exerted by body x on body y is equal in amount but in the opposite direction, in

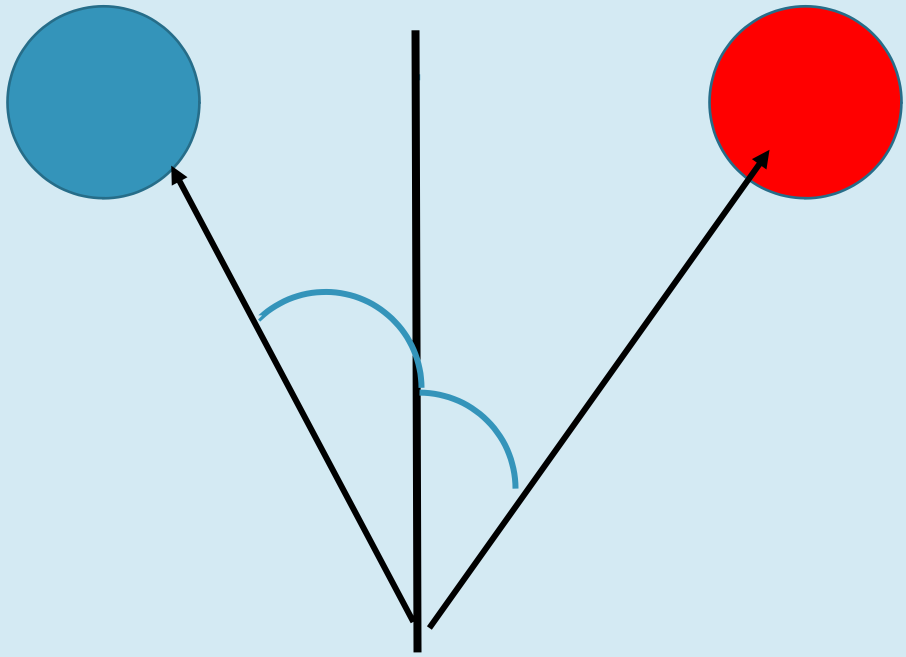
other words, a body gains as much momentum as it loses, but in the other direction. We can also say that the total change in momentum is zero, bringing us back to the law of conservation of momentum.

v. Momentum in two dimensions

Collisions can be of two types: straight line collisions and collisions at an angle to the x or y-axis. If bodies travel in straight lines, then they either have horizontal or vertical momentum. But if they have two-dimensional motion, then their momentum is also split into x and y components. When a body has momentum in two dimensions, then momentum in the same direction is equal before and after collisions.



After collision



The diagram shows a snooker ball traveling in the vertical direction. Before colliding, it has vertical momentum given by $m v_{y.}$. when it collides with another ball, it transfers some of its momentum to it. After hitting each other, both balls move off in different directions (here the angle is measured from the y axis). We find the total momentum in the x and y-axis after the collision:

$$momentum \in the\, y\, direction = m_1 v_y + m_2 v_y$$

$$momentum \in the\, x\, direction = m_1 v_x + m_2 v_x$$

Before colliding:

$$momentum \in the\, x\, direction = 0$$

$$momentum \in the\, y\, direction = m_1 u_y$$

Equating momentum in similar directions gives us:

$m_1 u_y = m_1 v_y + m_2 v_y$

And

$$m_1 v_x + m_2 v_x = 0$$

Chapter 6: Matter, density, and pressure

We are familiar with the three states of matter: solids, liquids, and gases. For a given amount of matter, density and volume vary greatly when external conditions like temperature and pressure are changed. The mass, however, does not change. This means that switching between different states does not change the mass, but the volume changes, resulting in different densities. Given that the mass remains the same, the greater the change in volume, the more the reduction in density. So if we take an ice cube and turn it into water vapor, the volume increases by a large amount. The density of the ice cube is considerably reduced. If we heat it until it liquefies, then the density decreases by a lesser amount.

First, let's look at the three states of matter:

Solids

- Particles vibrate in their fixed positions. Restricted movement except for slight collisions with each other.

- Particles are held together by strong forces of attraction.
- High melting and boiling points because of strong intermolecular (Van der Waals) forces of attraction.
- They have a fixed volume: they cannot reduce intermolecular distances to occupy the shape of their containers (you cannot fit a wooden box into a glass)
- They have the highest density out of all three states

Liquids

- Particles are arranged in random order and they can slide past each other.
- Intermolecular forces are stronger than those of solids.
- Melting and boiling points are higher than those of gases but lower than those of solids
- They can take up the shape of their containers but the volume does not change (liquids are slightly compressible)

Gases

- Particles are far apart from each other. The molecules can move freely in any direction.
- Weakest intermolecular forces of attraction
- Lowest melting and boiling points
- Highly compressible: the volume of a gas can be greatly reduced.

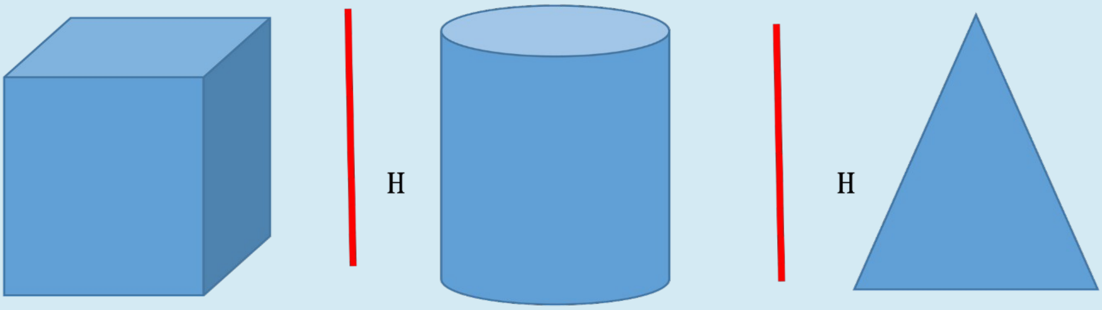
i. Pressure

Pressure is defined as the force acting per unit area. This is written as:

$$pressure = \frac{force}{area}$$

The unit is Pa (Nm^{-2}¿. If we hold the force constant, the area increases the pressure decreases. So pressure is inversely proportional to area. This can be proven through a simple example. Wear a pair of heels and stand on a muddy surface. You'll see a deep depression form. Now put on flat shoes and again stand on the muddy surface. The depression formed this time will have a smaller depth, showing that the pressure has been reduced. The force (your weight is constant) but the surface area increases, reducing the pressure exerted.

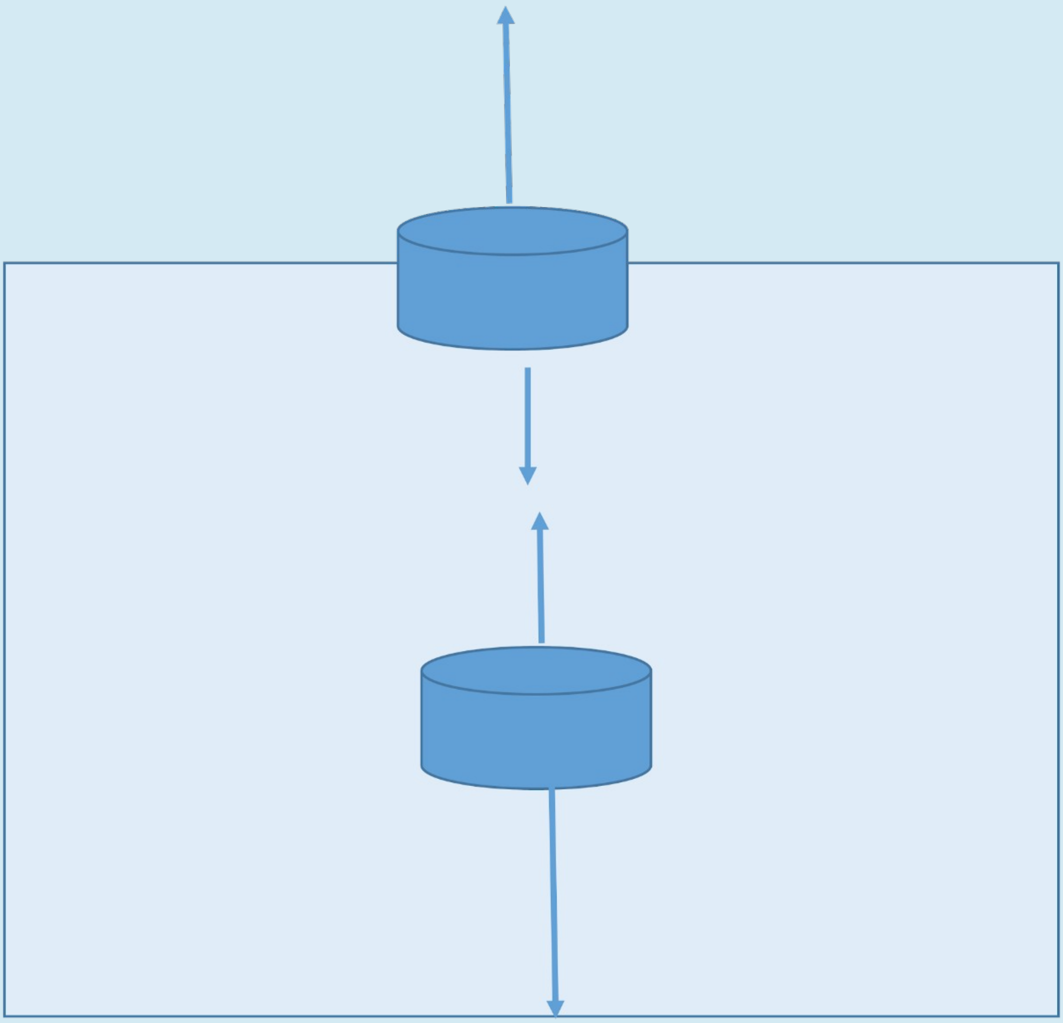
When we keep the area constant and vary the force, the pressure increases proportionally. Thus, pressure is directly proportional to force and inversely proportional to area.



SINCE HEIGHT IS THE SAME, PRESSURE IS THE SAME. SHAPE AND SIZE DOESN'T MATTER

ii. Upthrust and Archimedes' principle

UPTHRUST IS GREATER THAN THE WEIGHT, SO THE BLOCK FLOATS



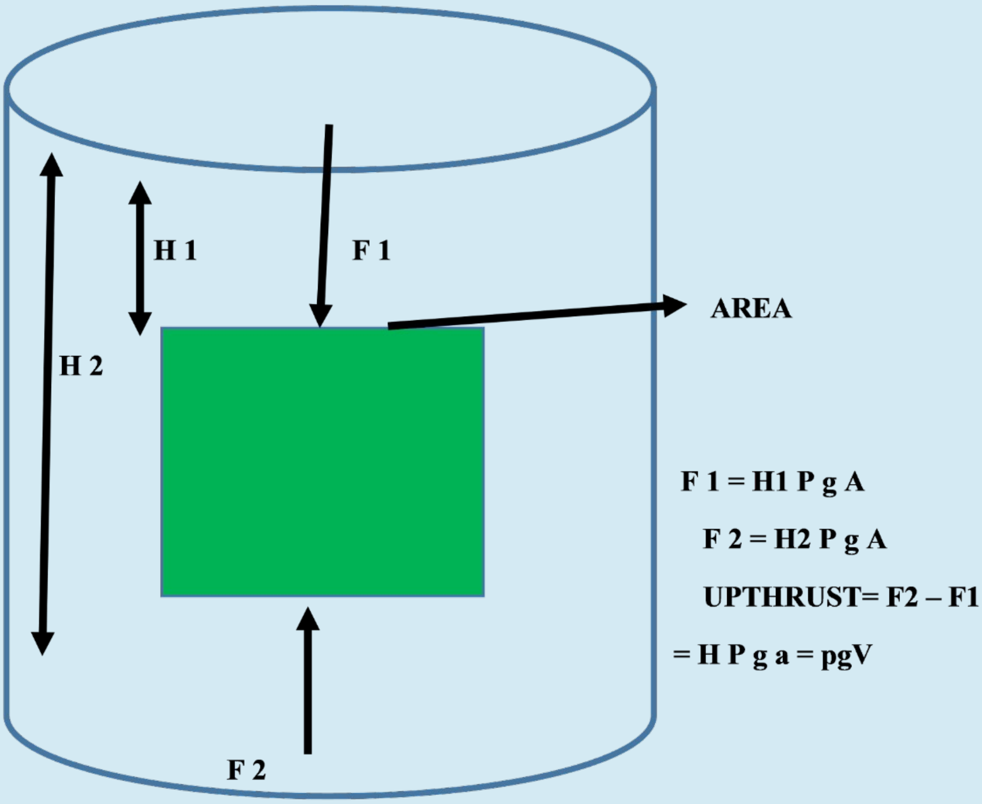
Weight is greater than the upthrust, so it sinks

A cube placed in a beaker would have pressure acting on all four surfaces as pressure acts in all directions. If we consider any two surfaces opposite to each other (one being the top surface and the other the bottom surface) we find that the pressure exerted on the bottom surface is more than that on the top surface. Larger pressures give rise to larger forces, resulting in

a net upward force on the bottom surface. This force is known as **upthrust**.

Archimedes' principle states that upthrust is equal to the weight of the liquid displaced.

Any solid would displace a volume of liquid equal to its volume. The force exerted by this liquid on the lower surface is equal to the object's weight. This can be proved in the following way:



The pressure on the top surface is given by P (top) = $h_1 pg$

Since pressure is transmitted equally throughout a liquid, the total pressure on the lower surface is given by P (bottom) = $h_2 ph + h_1 ph$

Subtracting the two gives us the pressure on the bottom surface:

$$= (h¿¿2\,ph + h_1\,ph) - h_1\,pg¿ = h_1\,pg$$

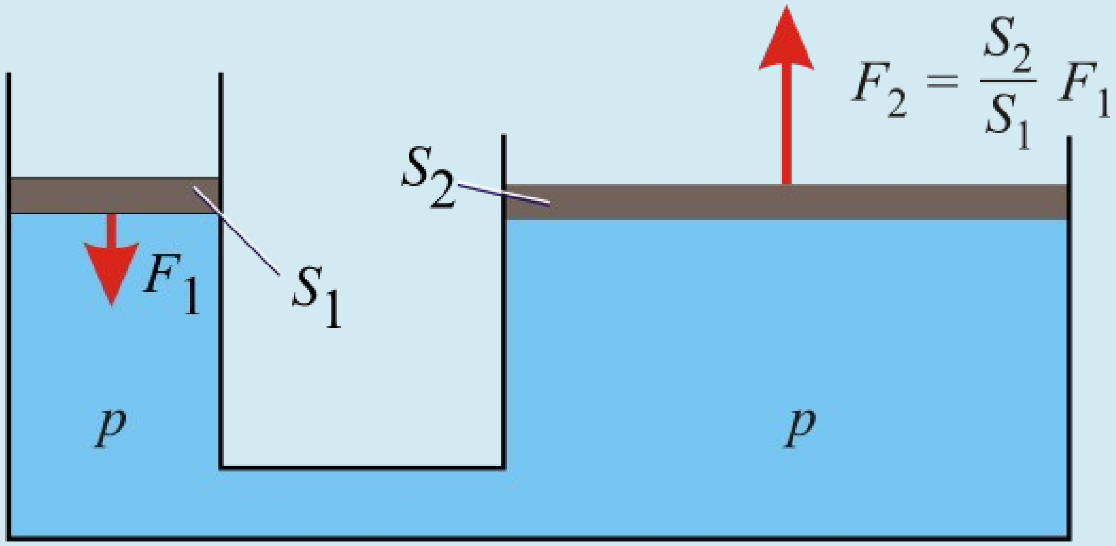
To get the force, we simply multiply the pressure with the area.

$$= h_1\,pg \times A = pVg$$

iii. Pressure transmission in liquids: The Hydraulic Press

The hydraulic press is a device that makes use of an important characteristic of incompressible liquids: pressure is transmitted uniformly. However, if we use a liquid of low compressibility, then the effectiveness of the hydraulic press is greatly reduced. Usually, liquids like water or oil are used.

How does the hydraulic press work?



A simple hydraulic press consists of a container with a raised opening on each side, as shown in the diagram. Two movable pistons of different surface areas are fitted on either side of the cylinder, which is then filled with either oil or water. Note that this is a simplified version of the hydraulic press. The ones used in cars are much narrower.

When a force is applied to the first piston, the piston exerts a pressure p_a on the oil underneath. This pressure is transmitted equally throughout the liquid. This transmission results in water exerting a pressure p_b on the second piston. p_a is equal top_b. The force, however, is proportional to the surface area of the piston. Piston B has twice the surface area, so the force is also doubled.

An important thing to note is that the hydraulic press follows the principle of conservation of energy. Energy transformed is equal to work

done. In moving piston A downwards, F_a moves a distance of D_a. The work done is given by:

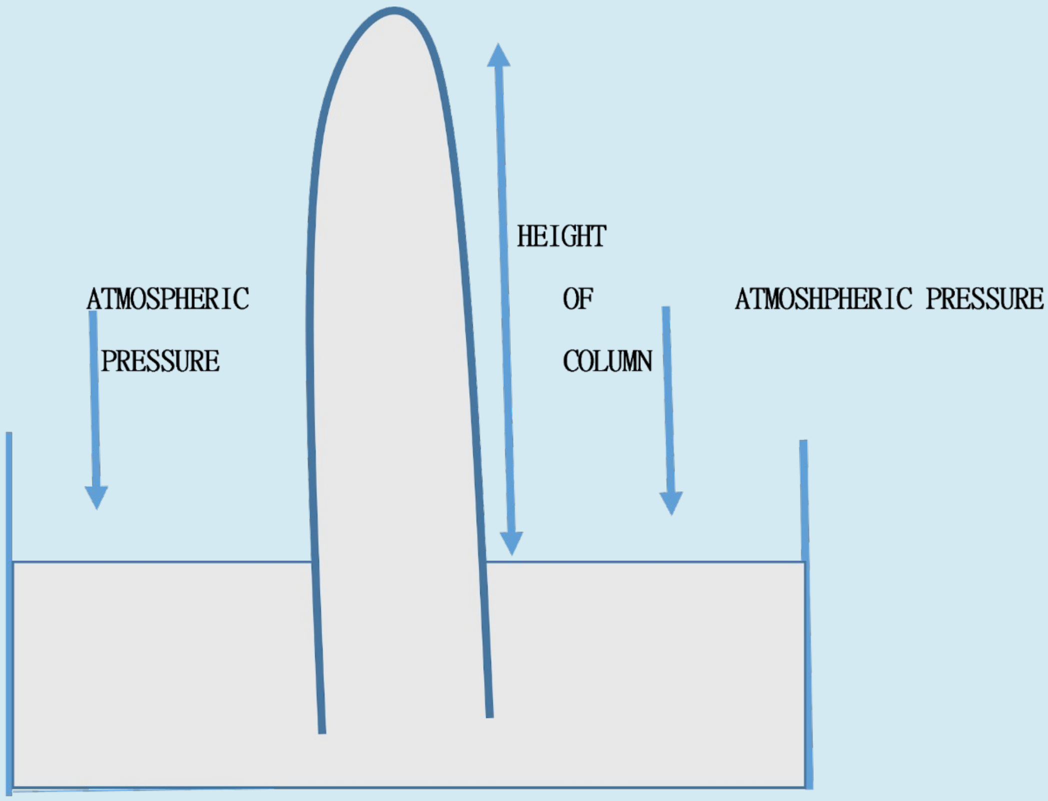
$= F_a \times D_a$

Piston B moves a relatively smaller distance. The force exerted on piston B is larger but the distance moved is smaller. So the product remains the same.

$F_b \times D_b = F_a \times D_b$

Measuring atmospheric and gas pressure.

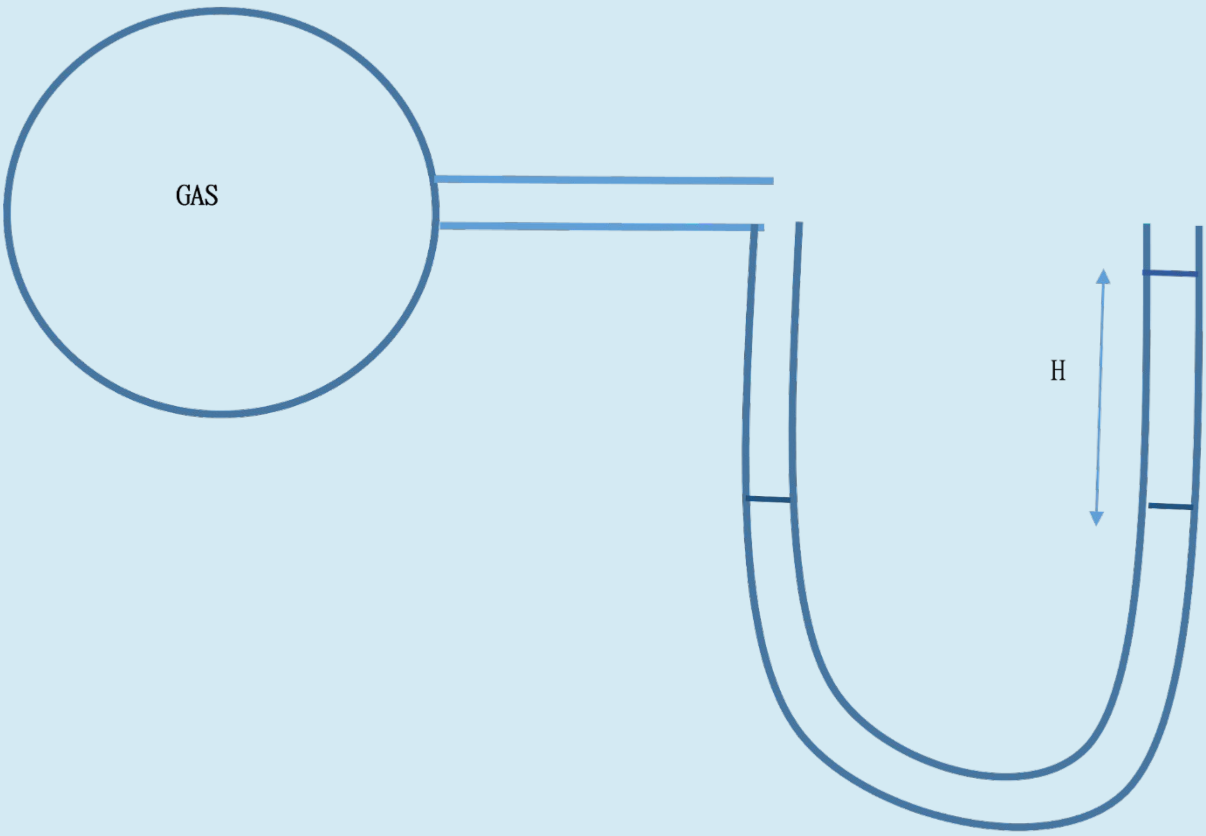
Atmospheric pressure, which is 1.01325 $\times 10^6$ Pa, can be measured using a device known as a barometer, which consists of an inverted tube placed in a beaker. The tube is filled with mercury, which is used because of its high density. If water, or any other liquid, was used instead of mercury, then the liquid level in the inverted tube would have risen by a large amount, requiring a very long tube.



At atmospheric pressure, the liquid column has a length of 760 mm or 76 cm. Point B shown in the diagram is the point of equilibrium, where the atmospheric pressure is equivalent to the pressure exerted by the liquid column above point B. Changing atmospheric conditions changes the length of the liquid column. For example, in areas of high altitude, where the pressure is lower than atmospheric pressure, the length of the mercury column decreases. The opposite is true in areas of high pressure.

Manometers

PRESSURE DIFFERENCE IS GIVEN BY H



Manometers are used to measure differences in gas pressure. A typical manometer consists of a U- shaped tube, with one end open and the other connected to a gas supply. Before connecting the gas supply, the liquid level is even on both sides of the tube. However, the level on one side would either rise or fall, depending on the difference between gas and atmospheric pressure. If gas pressure is more than atmospheric pressure, then the liquid level on the side open to the atmosphere would rise. If atmosphere pressure is more than gas pressure, then the liquid level on the gas side would rise.

The above situation can be summarized as follows:

$p_g = p_{atm} + hpg$

➢Chapter 6: Deformations

What happens when you stretch a spring? As you pull it downwards, its length increases. When you release it, the spring returns to its original length. But what if you apply a lot of force in the first place? You'll notice that upon removing the force, the spring does not return to its original length. Moreover, if you further apply force, the spring becomes easy to stretch, indicating that it has been permanently deformed. In this chapter, we'll be studying the behavior of different materials when different amounts of forces are applied.

i. Hooke's law and the limit of proportionality

Extension is defined as the difference between the stretched and unstretched lengths. The extension of a spring depends on its stiffness, which is called the spring constant.

The spring constant is a measure of the amount of force required to increase the length of a spring by 1 m.

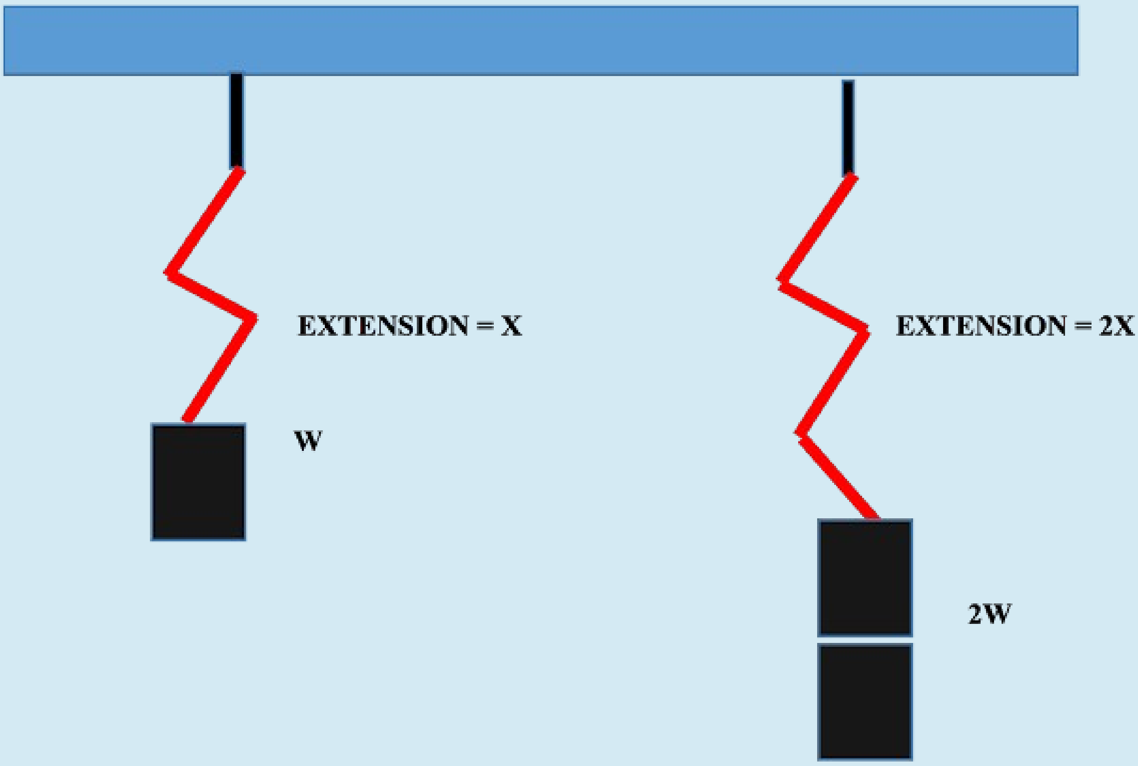
The spring constant of a spring does not change as long as a spring does not exceed its limit of proportionality. Before this point, the more the force applied the more the extension. Force is directly proportional to extension. This relationship can be expressed as:

$f = ek$

Where f is the applied force

E is the extension in m

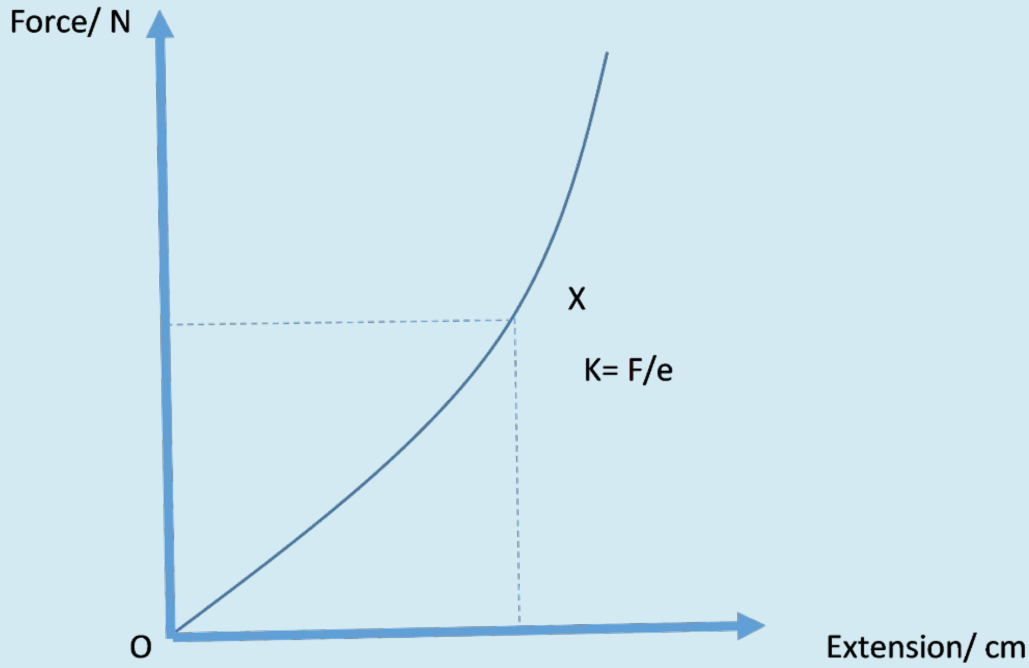
K is the spring constant in N/m



Hooke’s law

Hooke’s law states that the force applied is directly proportional to extension until a spring reaches its limit of proportionality. In other words, force and extension increase in the same ratio till the limit of proportionality.

If the force is no longer proportional to extension, the spring constant changes. This means that the spring either becomes too stiff or it becomes easy to extend. A force-extension graph can be plotted to show the change in the spring constant.



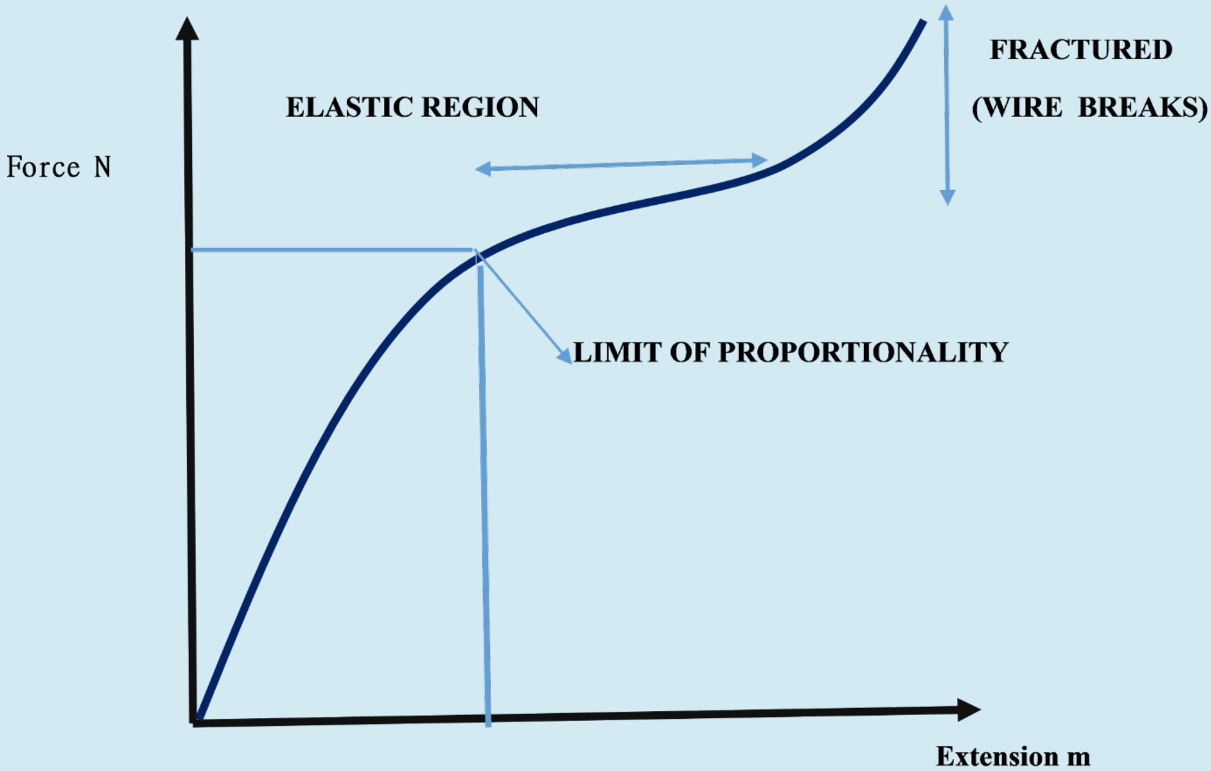
As long as the graph shows an upward sloping straight line Hooke's law is obeyed. When the graph starts to curve upwards or downwards, the spring is no longer obeying Hooke's law.

Experiment to determine the limit of proportionality

To find a spring's limit of proportionality, measure the initial length of a spring using a meter rule. Attach a mass and note the stretched length. Repeat the process until you obtain six

readings. Note that the masses attached have to be identical to ensure that the force increases by the same amount each time. Plot the readings on a graph. The point on the graph where the straight line starts to curve is the limit of proportionality.

ii. Elastic limit and deformations



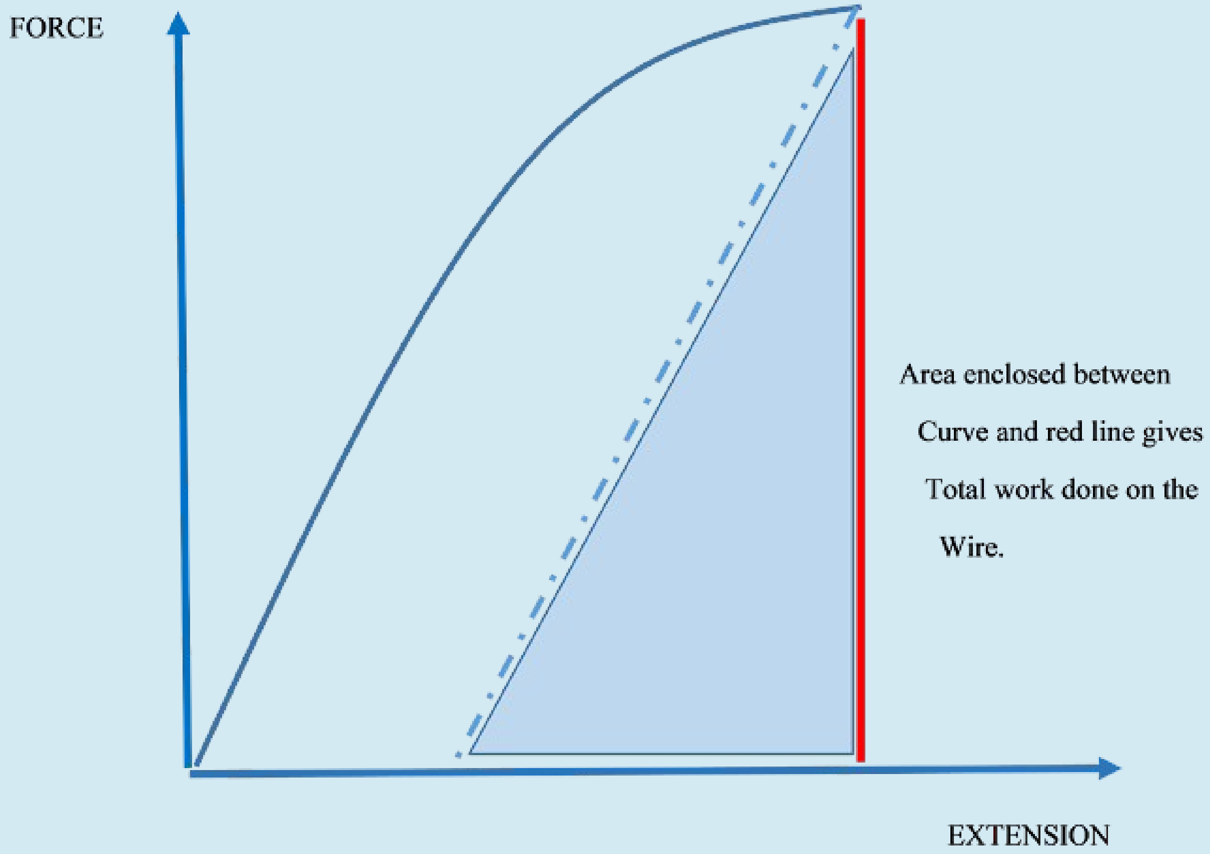
An object (such as a spring) undergoes deformations under the influence of a force. But an object may or may not return to its original length when the force is removed. The ability to return to its original length depends on the extent of deformation, which can be shown by the elastic limit. But before moving on to the

elastic limit, it is important to understand the two main types of deformations.

Elastic deformation takes place when the deformed object can return to its original measurements. For example, a spring measures 10 cm. You stretch it and then let it go. If the spring returns to exactly 10 cm, it was elastically deformed.

On the contrary, if the applied force is large enough to deform the spring permanently, then it undergoes plastic deformation. When a spring is plastically deformed, the final dimensions are always more than the original ones.

The elastic limit is the point that sets the boundary between elastic and plastic deformation. It is the maximum value of stress (discussed below) for which a spring can be elastically deformed.



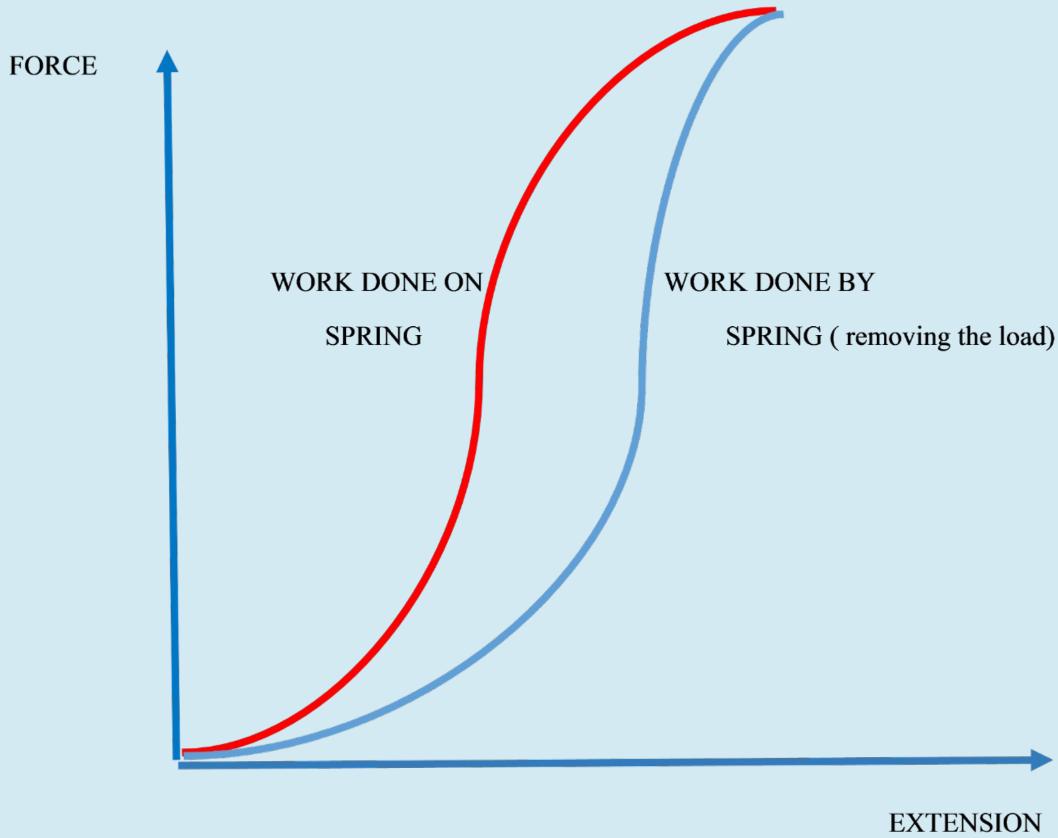
Usually, but not always, the elastic limit of a spring is the same as the limit of proportionality. For springs where the two points are not the same, the elastic limit lies just above the limit of proportionality.

The graphs show two springs that undergo elastic and plastic deformation respectively. For spring A, a line parallel to the original slope can be drawn back to the origin, showing the spring is able to return to its original length.

When a similar line is drawn for spring B, the line meets another point on the x-axis, which means that it has been permanently deformed.

Unusual curves

The force-extension graph shows a curve for a particular spring. Note that the graph is a curve, not a straight line. This means that the spring has exceeded its limit of proportionality. Since the elastic limit is found just above the LOP, the spring has also exceeded its elastic limit. However, drawing a similar curve back to the origin shows that the spring hasn't been permanently deformed. Thus, it can be concluded that spring has crossed its elastic limit but it hasn't been permanently deformed. This is an unusual case but is only possible for a small value of extension.



Calculating the elastic potential energy from the area underneath a force-extension graph

The area underneath a force-extension graph can be used to calculate the work done on a spring as well as the work done by a spring.

$$\text{Work done} = \text{force} \times \text{distance moved}$$

$$= \text{force} \times \text{extension}$$

We also know that work done is equal to energy transformed. So the work done on a spring gives us the elastic potential energy transferred to it. On the other hand, the work done by a spring gives us the energy it loses as thermal energy.

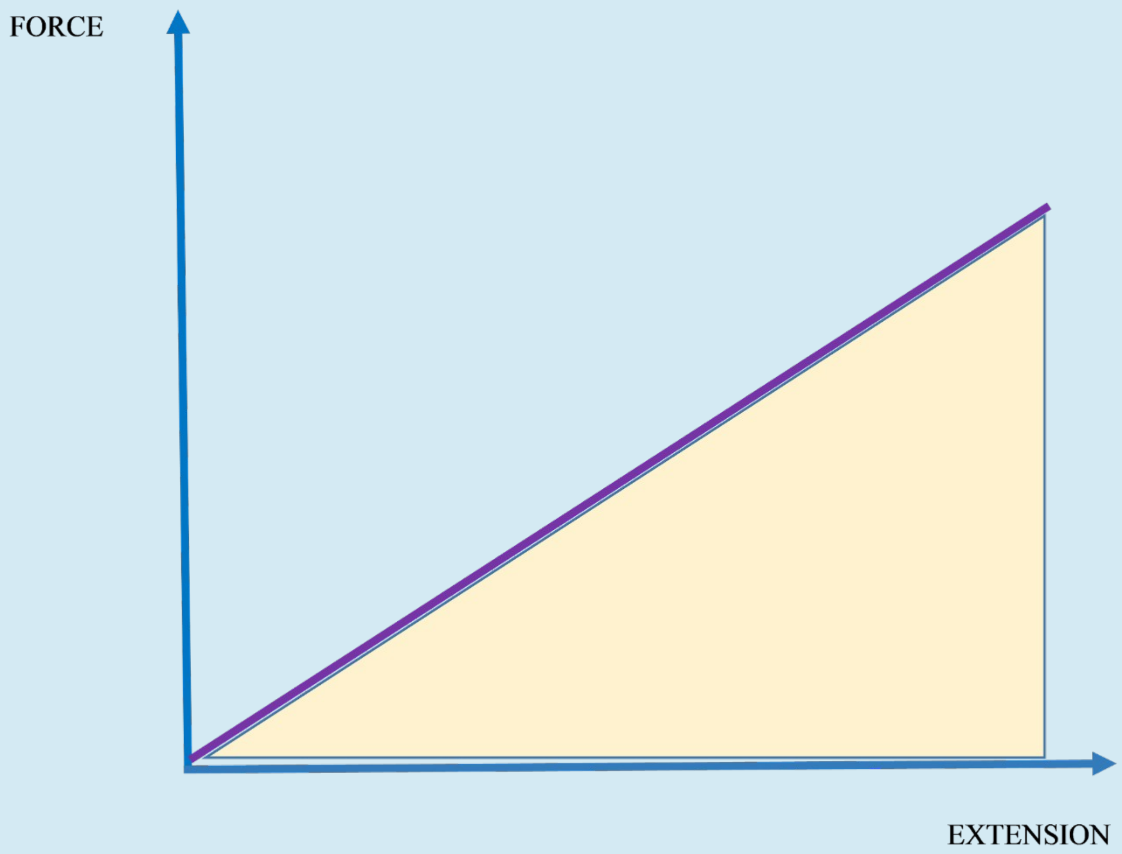
For straight-line graphs the area is given by the area of a triangle:

$$area = \frac{1}{2} \times base \times height$$

$$\text{¿} \frac{1}{2} force \times extension$$

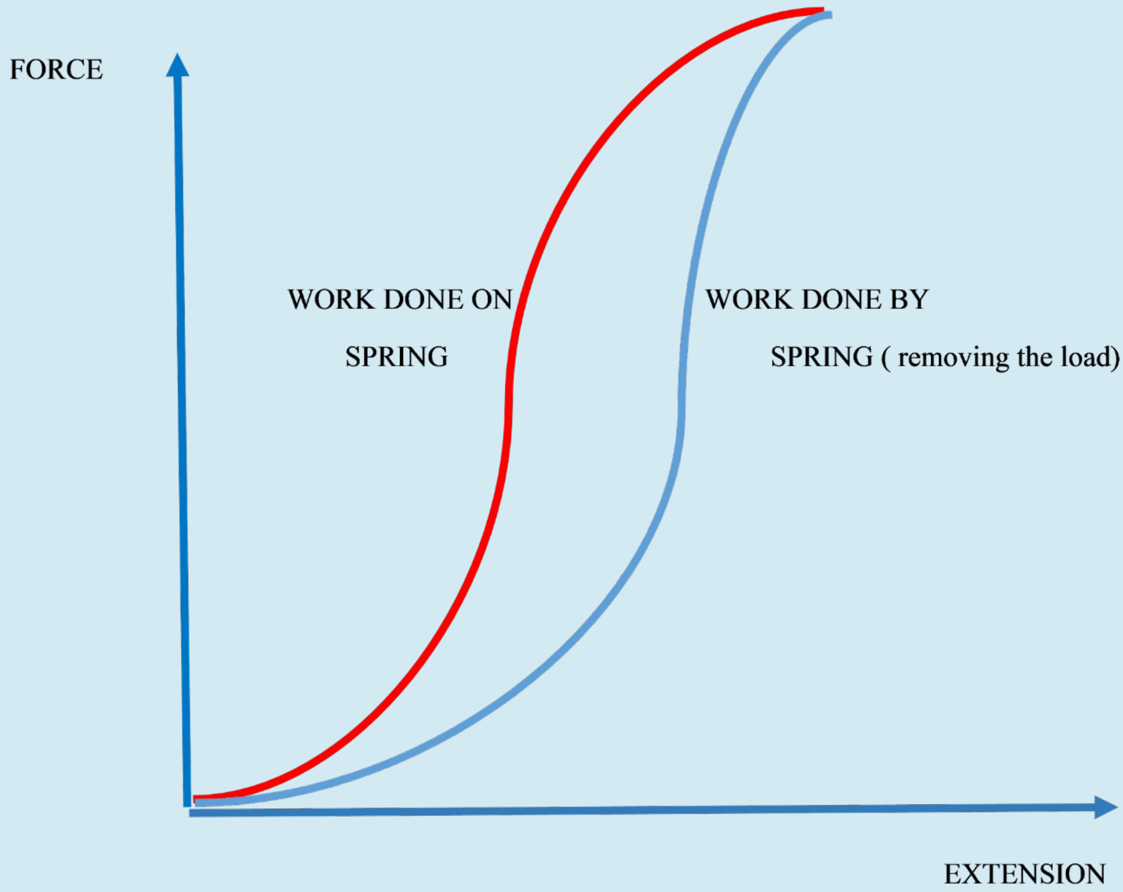
$$\text{¿} \frac{1}{2} e^2 k$$

For curves, the area can be divided into equal square units. The area of one unit can be calculated and multiplied by the total number of divisions. This gives an estimate of the energy stored or released.



Example of energy calculations

The following diagram shows two curves: the upper one for loading and the lower one for unloading. The area under the upper curve shows the total work done on the spring. Hence, it represents the elastic potential energy stored in it. The area under the lower curve represents the work done by the spring. This is equivalent to wasted energy. The shaded area between the two curves gives us the energy retained by the spring.



Young's modulus

We know that applying a force on a spring produces extension. We also know that for the same amount of force applied, the extension depends on the stiffness, which is known as the spring constant.

We are now going to express the load and extension as rates and ratios. These can then be used to calculate the young's modulus, which is another way of measuring the stiffness of a spring.

Stress

Stress is defined as the force acting per unit area. Stress is given by:

$$\text{¿}\frac{force}{area}(Nm^{-2})$$

Note that stress has the same unit as pressure. The only difference between stress and pressure is the type of force. While the former is used for compressive forces, the latter refers to tensile forces.

Strain

Strain is the ratio of the extension and the original length of a spring. Strain tells us the change in length per unit of the original length. This is given by:

$$\text{¿}\frac{extension}{original\,length}$$

Stress and strain combine to give us the young’s modulus of a material.

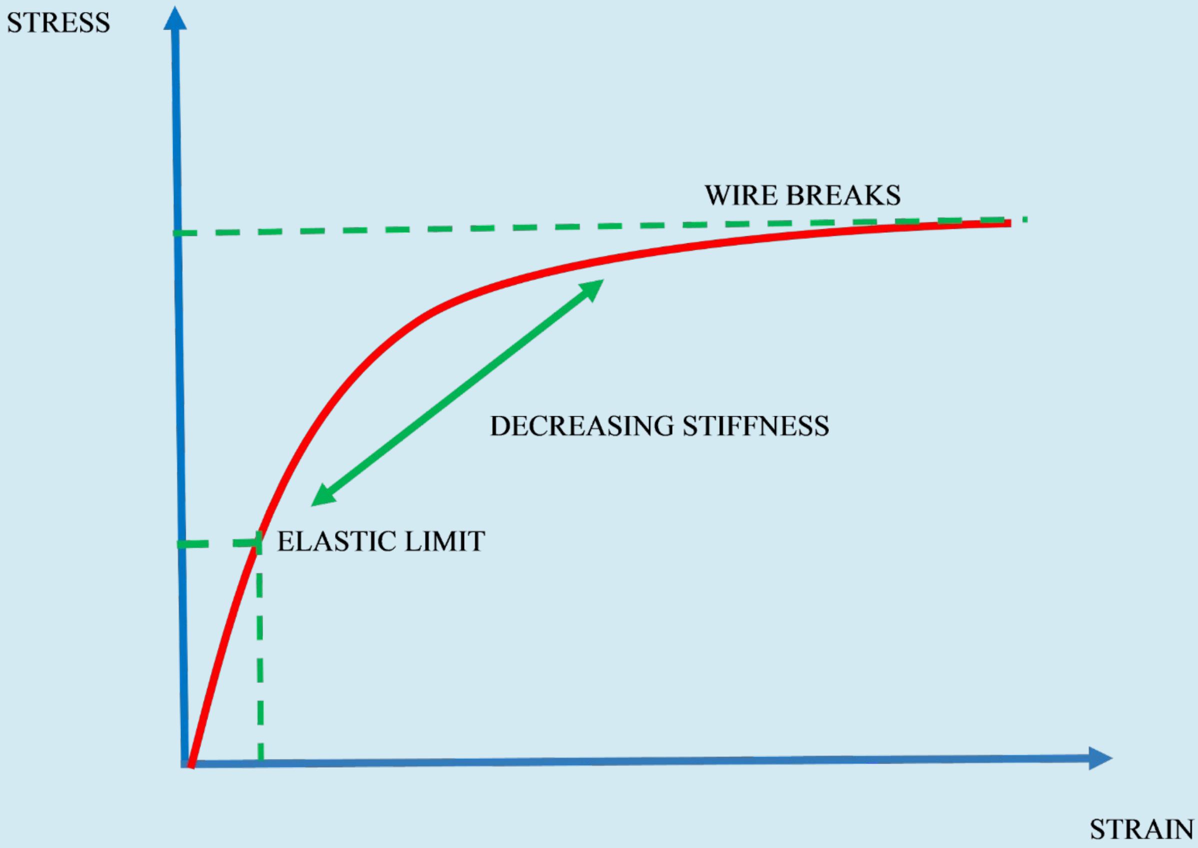
$$young's\,modulus=\frac{stress}{strain}$$

$$\text{¿}\frac{\frac{force}{area}(Pa)}{\frac{extension}{original\,length}}$$

$$\text{¿}\frac{FL}{AE}(Pa)$$

The value of young’s modulus for every material is different and constant. It only changes when the temperature or pressure changes. Changing the length or area of a spring does not change

the young’s modulus because if one variable changes, the other changes by the same amount.



Types of materials and their stress-strain graphs.

Materials are either classified as brittle or ductile. Brittle materials cannot be bent into different shapes; they break immediately when a lot of force is applied. Examples include ceramics, steel, and iron. These materials have high values of young’s modulus, which means that their stress-strain graphs are very steep. In addition, their graphs are represented by straight lines, curving only at the endpoint. This shows that their value of stiffness does not change.

On the other hand, ductile materials can be bent after a large force is applied. Their graphs are less steep than those of brittle materials. Also, the graph has a large curved part, indicating that the stiffness has changed.

➢Chapter 7: Work, Energy, and Power

In everyday life, anything that requires physical exertion is referred to as work. Holding a book, pushing against a wall, and trying to move a heavy grocery trolley are referred to as doing work, but no work is done.

In physics, work means moving a distance in the direction of the applied force. In the above examples, forces are acting on the objects but no distance is being moved in their directions. When you hold a book, you are applying an upward supporting force, but the book is stationary so no work is being done. Similarly, the book's weight acts in the downward direction but the book is not displaced downwards.

Work done is calculated by multiplying the applied force with the distance moved in the direction of the force. The unit is J or Nm.

$work\,done = force \times distance$

¿ $F \times S$

Even though work done involves vector quantities, it is a scalar quantity. We can express it in terms of its base units:

¿ Nm

¿ $Kgm\,s^{-2}.m$

¿ $kg\,m^{2}\,s^{-2}$

Energy transferred and work done

Energy is transferred whenever work is done. Energy can be transferred from any form (gravitational potential to kinetic and kinetic to thermal) but in every case the total energy conversion represents the total work done.

For example, a skydiver jumps from a helicopter. His or her gravitational potential energy is converted to kinetic energy, along with thermal energy in work done against air resistance.

In this case, work done is given by

Work = kinetic energy + thermal energy

Work done for forces with components

Forces can be resolved into their horizontal and vertical components to find the work done in a particular direction.

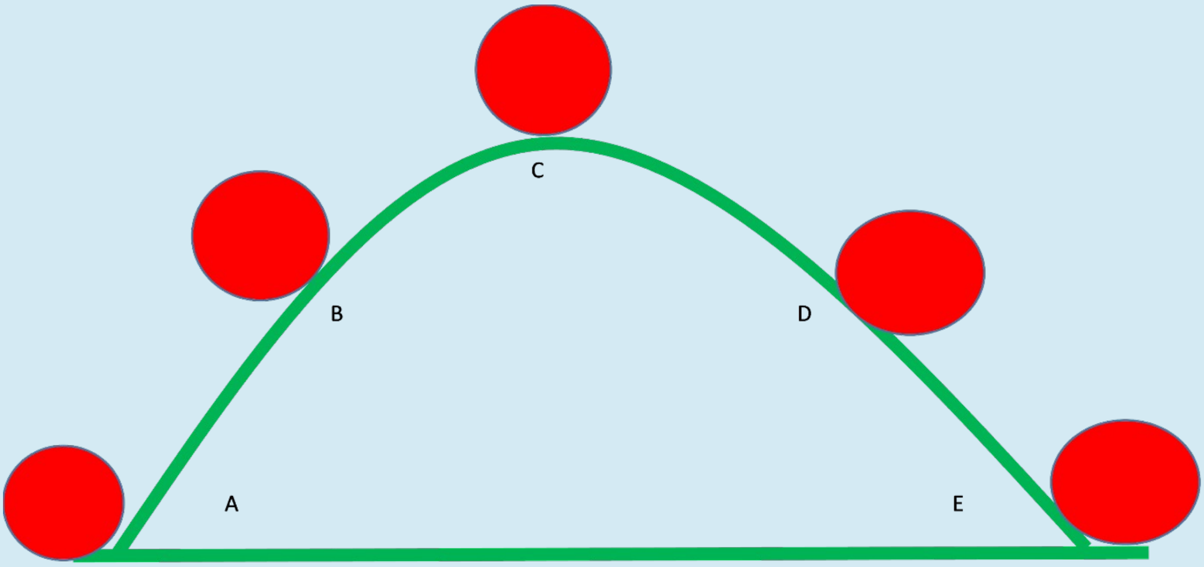
A 50 N force is acting at an angle of 30 degrees to the horizontal will have the following components:

Vertical direction = 50 sin 30

Horizontal direction = 50 cos 30

Work done in either direction can be found by resolving the displacement of the 50 N force and then multiplying like components together.

Main Energy conversions



GPE to kinetic energy is the most important energy conversion that takes place around us. A body falling vertically downwards loses GPE at every point and gains kinetic energy. Similarly, a body traveling upwards gains gravitational potential energy. Ignoring the effects of air resistance, the gain in kinetic energy is exactly equal to the loss in G.P.E.

Using this concept we can equate the two changes together:

Gain in kinetic energy = loss in GPE

$\Delta \frac{1}{2} m v^2 = \Delta mgh$

Efficiency

If there is no air resistance present then energy transfers are perfect: all the energy is converted from one useful form to another. In this case, the system is said to be 100 % efficient. However, if

friction and air resistance are present, a great deal of energy is wasted as thermal and sound energy. Then we say that the efficiency of the system is reduced.

Efficiency is the ratio of the useful energy output to the total energy input.

Efficiency = $\frac{useful\,energy\,output}{total\,energy\,input} \times 100.$

Useful energy output = input – wasted energy

If useful output = wasted energy the system is said to be 50 % efficient.

Principle of conservation of energy

Regardless of the type of energy conversion, the total amount of energy before a conversion is equal to the total amount after the conversion. This means the energy itself is not destroyed or created.

The idea of energy simply changing forms is expressed by the principle of conversion of energy:

Energy can neither be created nor destroyed, but transformed from one form to the other.

However, it is the magnitude of useful transformation that determines the efficiency value of a system.

Power

Sometimes we need to compare the energy conversions of two bodies. The work done is the same (two boys climbing a set of stairs or two filament lamps converting chemical energy to electrical energy) but the time taken is different. In order to make a comparison, we find the rate of work done.

Power is defined as the rate of energy conversion or work done.

$$\text{Power} = \frac{work\,done}{time\,taken}$$

¿ $\frac{force \times distance}{time\,taken}$

¿ $force \times velocity$

To use the above equation, the velocity has to be constant and the force should be the one that is causing the body to do work

Moreover, to find the work done by a gas we use the following equation:

¿ $pressure \times volume$

¿ $pressure \times area \times length$

¿ $force \times distance$

Therefore, we arrive at the same equation for work done.